CHAPTER 7

Heat Transfer

Heat transfer is the movement of energy from one point to another by virtue of a difference in temperature. Heating and cooling are manifestations of this phenomenon, which is used in industrial operations and in domestic activities. Increasing energy costs and in some cases inadequate availability of energy will require peak efficiency in heating and cooling operations. An understanding of the mechanisms of heat transport is needed in order to recognize limitations of heating and cooling systems, which can then lead to adoption of practices that circumvent these limitations. In industrial and domestic heating and cooling, energy-use audits can be used to determine total energy use and the distribution within the process, to identify areas of high energy use, and to target these areas for energy conservation measures.

7.1 MECHANISMS OF HEAT TRANSFER

Heat will be transferred from one material to another when there is a difference in their temperature. The temperature difference is the driving force which establishes the rate of heat transfer.

7.1.1 Heat Transfer by Conduction

When heat is transferred between adjacent molecules, the process is called conduction. This is the mechanism of heat transfer in solids.

7.1.2 Fourier's First Law of Heat Transfer

According to Fourier's first law, the heat flux, in conduction heat transfer, is proportional to the temperature gradient.

$$\frac{q}{A} = -k\frac{dT}{dx} \tag{7.1}$$

In Equation (7.1), q is the rate of heat flow, and A is the area through which heat is transferred. A is the area perpendicular to the direction of heat flow. The expression q/A, the rate of heat transfer per unit area, is called the heat flux. The derivative dT/dx is the temperature gradient. The negative sign in Equation (7.1) indicates that positive heat flow will occur in the direction of decreasing temperature. The parameter k in (7.1), the thermal conductivity, is a physical property of a material. Values of the thermal conductivity of common materials of construction and insulating materials and of food products are given in Appendix Tables A.9 and A.10, respectively.

7.1.3 Estimation of Thermal Conductivity of Food Products

The thermal conductivity of materials varies with the composition and, in some cases, the physical orientation of components. Foods, being of biological origin possess highly variable composition and structure, therefore, k of foods presented in the tables is not always the same for all foods in the category listed. The effect of variations in the composition of a material on values of the thermal conductivity, has been reported by Choi and Okos (1987). Their procedure may be used to estimate k from the composition. k is calculated from the thermal conductivity of the pure component k_i and the volume fraction of each component, X_{vi}. An important assumption used in this estimation procedure is that the contribution of each component to the composite thermal conductivity is proportional to the component volume fraction as follows:

$$k = \sum (k_i X_{vi}) \tag{7.2}$$

The thermal conductivity in W/(m · K) of pure water (k_w), ice (k_{ic}), protein (k_p), fat (k_f), carbohydrate (k_c), fiber (k_{fi}), and ash (k_a) are calculated at the temperature, T in °C, using Equations (7.3) to (7.9), respectively.

$$k_w = 0.57109 + 0.0017625\,T - 6.7306 \times 10^{-6}\,T^2 \tag{7.3}$$

$$k_{ic} = 2.2196 - 0.0062489\,T + 1.0154 \times 10^{-4}\,T^2 \tag{7.4}$$

$$k_p = 0.1788 + 0.0011958\,T - 2.7178 \times 10^{-6}\,T^2 \tag{7.5}$$

$$k_f = 0.1807 - 0.0027604\,T - 1.7749 \times 10^{-7}\,T^2 \tag{7.6}$$

$$k_c = 0.2014 + 0.0013874\,T - 4.3312 \times 10^{-6}\,T^2 \tag{7.7}$$

$$k_{fi} = 0.18331 + 0.0012497\,T - 3.1683 \times 10^{-6}\,T^2 \tag{7.8}$$

$$k_a = 0.3296 + 0.001401\,T - 2.9069 \times 10^{-6}\,T^2 \tag{7.9}$$

The volume fraction X_{vi} of each component is determined from the mass fraction X_i, the individual densities ρ_i, and the composite density, ρ as follows:

$$X_{vi} = \frac{X_i \rho}{\rho_i} \tag{7.10}$$

The individual densities, in kg/m^3, are obtained using Equations (7.12) to (7.18), respectively, for water (ρ_w), ice (ρ_{ic}), protein (ρ_p), fat(ρ_f), carbohydrate (ρ_c), fiber (ρ_{fi}), and ash (ρ_a).

$$\rho = \frac{1}{[\sum (X_i/\rho_i)]} \tag{7.11}$$

$$\rho_w = 997.18 + 0.0031439\,T - 0.0037574\,T^2 \tag{7.12}$$

$$\rho_{ic} = 916.89 - 0.13071\,T \tag{7.13}$$

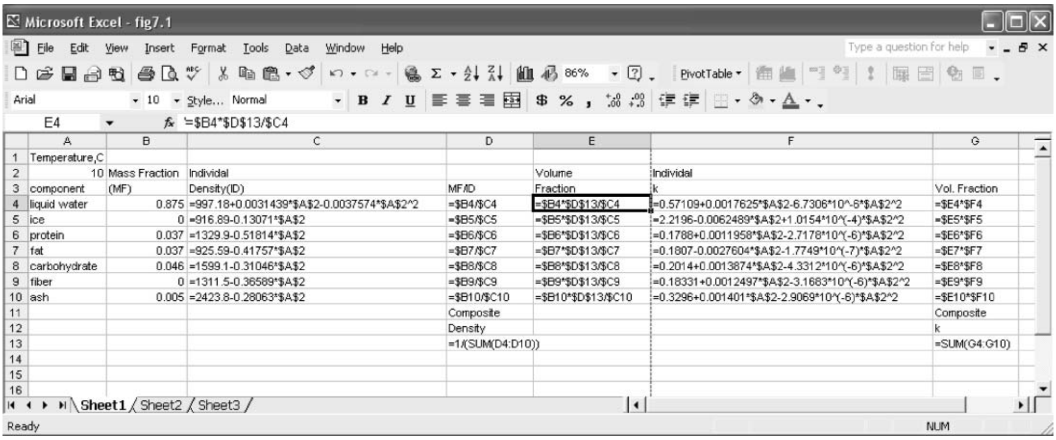
$$\rho_p = 1329.9 - 0.51814\,T \tag{7.14}$$

$$\rho_f = 925.59 - 0.41757\,T \tag{7.15}$$

$$\rho_c = 1599.1 - 0.31046\,T \tag{7.16}$$

$$\rho_{fi} = 1311.5 - 0.36589\,T \tag{7.17}$$

$$\rho_a = 2423.8 - 0.28063\,T \tag{7.18}$$



	A	B	C	D	E	F	G
1	Temperature,C						
2	10	Mass Fraction	Individal		Volume	Individal	
3	component	(MF)	Density(ID)	MF/ID	Fraction	k	Vol. Fraction
4	liquid water	0.875	=997.18+0.0031439*$A$2-0.0037574*$A$2^2	=$B4/$C4	=$B4*$D$13/$C4	=0.57109+0.0017625*$A$2-6.7306*10^-6*$A$2^2	=$E4*$F4
5	ice	0	=916.89-0.13071*$A$2	=$B5/$C5	=$B5*$D$13/$C5	=2.2196-0.0062489*$A$2+1.0154*10^(-4)*$A$2^2	=$E5*$F5
6	protein	0.037	=1329.9-0.51814*$A$2	=$B6/$C6	=$B6*$D$13/$C6	=0.1788+0.0011958*$A$2-2.7178*10^(-6)*$A$2^2	=$E6*$F6
7	fat	0.037	=925.59-0.41757*$A$2	=$B7/$C7	=$B7*$D$13/$C7	=0.1807-0.0027604*$A$2-1.7749*10^(-7)*$A$2^2	=$E7*$F7
8	carbohydrate	0.046	=1599.1-0.31046*$A$2	=$B8/$C8	=$B8*$D$13/$C8	=0.2014+0.0013874*$A$2-4.3312*10^(-6)*$A$2^2	=$E8*$F8
9	fiber	0	=1311.5-0.36589*$A$2	=$B9/$C9	=$B9*$D$13/$C9	=0.18331+0.0012497*$A$2-3.1683*10^(-6)*$A$2^2	=$E9*$F9
10	ash	0.005	=2423.8-0.28063*$A$2	=$B10/$C10	=$B10*$D$13/$C10	=0.3296+0.001401*$A$2-2.9069*10^(-6)*$A$2^2	=$E10*$F10
11				Composite			Composite
12				Density			k
13				=1/(SUM(D4:D10))			=SUM(G4:G10)
14							
15							
16							

Figure 7.1 Spreadsheet program in Excel used to calculate the thermal conductivity of milk.

A Spreadsheet program in Excel shown in Fig. 7.1 may be used to calculate the thermal conductivity from the composition of a material.

To use this program, enter the temperature in cell A2 and the mass fraction of each specified component in B4 to B10. Excel will calculate required intermediate values and display the composite thermal conductivity in cell G13. Enter zero for a component whose mass fraction is not given.

Example 7.1. Calculate the thermal conductivity of lean pork containing 7.8% fat, 1.5% ash, 19% protein, and 71.7% water, at 19°C.

Solution:

The spreadsheet showing values of the component mass fraction in B4 to B10 and the composite thermal conductivity is shown in Fig. 7.2. The composite thermal conductivity is 0.4970 W/m · AK displayed in cell G13 in Fig. 7.2.

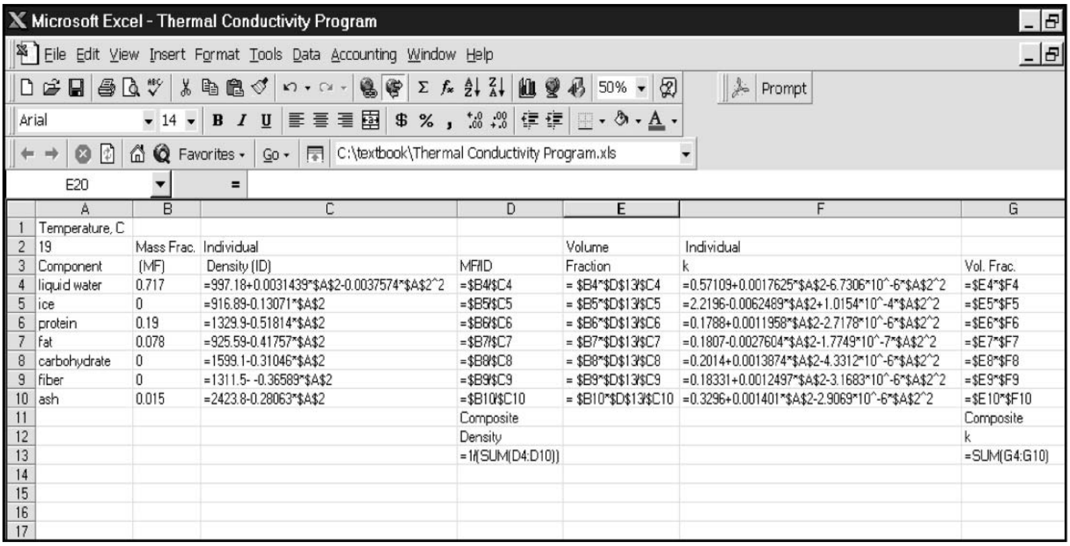
Example 7.2. Calculate the thermal conductivity of milk which contains 87.5% water, 3.7% protein, 3.7% fat, 4.6% lactose, and 0.5% ash, at 10°C.

Solution:

The spreadsheet program (Fig. 7.1) is used with the mass fraction of components and the temperature entered in the corresponding cells. Results are shown in Fig. 7.3.

The composite thermal conductivity is shown in cell G13 to be 0.5473 W/mAK.

Data on average composition of foods from USDA Handbook 8, which can be used to estimate thermophysical properties, is given in Appendix A.8.

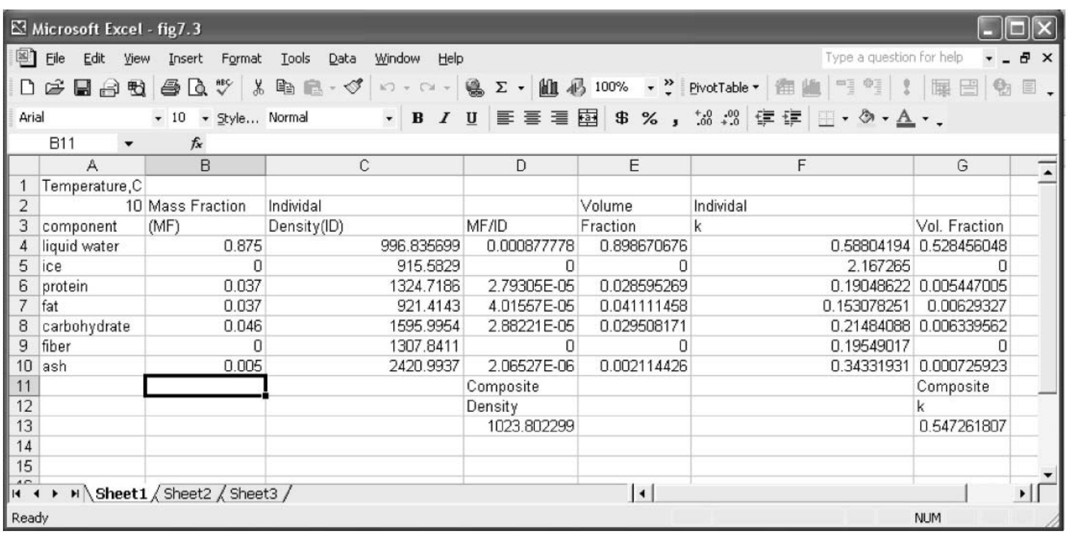


	A	B	C	D	E	F	G
1	Temperature, C						
2	19	Mass Frac.	Individual		Volume	Individual	
3	Component	(MF)	Density (ID)	MF/ID	Fraction	k	Vol. Frac.
4	liquid water	0.717	=997.18+0.0031439*$A$2-0.0037574*$A$2^2	=$B4/$C4	= $B4*$D$13/$C4	=0.57109+0.0017625*$A$2-6.7306*10^-6*$A$2^2	=$E4*$F4
5	ice	0	=916.89-0.13071*$A$2	=$B5/$C5	= $B5*$D$13/$C5	=2.2196-0.0062489*$A$2+1.0154*10^-4*$A$2^2	=$E5*$F5
6	protein	0.19	=1329.9-0.51814*$A$2	=$B6/$C6	= $B6*$D$13/$C6	=0.1788+0.0011958*$A$2-2.7178*10^-6*$A$2^2	=$E6*$F6
7	fat	0.078	=925.59-0.41757*$A$2	=$B7/$C7	= $B7*$D$13/$C7	=0.1807-0.0027604*$A$2-1.7749*10^-7*$A$2^2	=$E7*$F7
8	carbohydrate	0	=1599.1-0.31046*$A$2	=$B8/$C8	= $B8*$D$13/$C8	=0.2014+0.0013874*$A$2-4.3312*10^-6*$A$2^2	=$E8*$F8
9	fiber	0	=1311.5- -0.36589*$A$2	=$B9/$C9	= $B9*$D$13/$C9	=0.18331+0.0012497*$A$2-3.1683*10^-6*$A$2^2	=$E9*$F9
10	ash	0.015	=2423.8-0.28063*$A$2	=$B10/$C10	= $B10*$D$13/$C10	=0.3296+0.001401*$A$2-2.9069*10^-6*$A$2^2	=$E10*$F10
11				Composite			Composite
12				Density			k
13				=1/(SUM(D4:D10))			=SUM(G4:G10)
14							
15							
16							
17							

Figure 7.2 Spreadsheet program in Excel used to calculate the thermal conductivity of lean pork.

7.1.4 Fourier's Second Law of Heat Transfer

When the rate of heat transfer across a solid is not uniform (i.e., there is a difference in the rate at which energy enters and leaves a control volume), this difference will be manifested as a rate of



	A	B	C	D	E	F	G
1	Temperature,C						
2	10	Mass Fraction	Individal		Volume	Individal	
3	component	(MF)	Density(ID)	MF/ID	Fraction	k	Vol. Fraction
4	liquid water	0.875	996.835699	0.000877778	0.898670676	0.58804194	0.528456048
5	ice	0	915.5829	0	0	2.167265	0
6	protein	0.037	1324.7186	2.79305E-05	0.028595269	0.19048622	0.005447005
7	fat	0.037	921.4143	4.01557E-05	0.041111458	0.153078251	0.00629327
8	carbohydrate	0.046	1595.9954	2.88221E-05	0.029508171	0.21484088	0.006339562
9	fiber	0	1307.8411	0	0	0.19549017	0
10	ash	0.005	2420.9937	2.06527E-06	0.002114426	0.34331931	0.000725923
11				Composite			Composite
12				Density			k
13				1023.802299			0.547261807
14							
15							

Figure 7.3 Excel Spreadsheet showing the solution for Example 7.2.

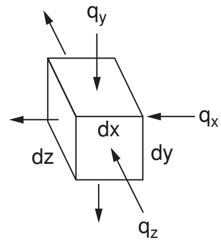


Figure 7.4 Control volume for analysis of heat transfer into a cube.

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change of temperature with time, within the control volume. This type of problem is called unsteady state heat transfer. Figure 7.4 shows a control volume for analyzing heat transfer into a cube with sides dx, dy, and dz. The following are heat balance equations across the control volume in the x, y and z directions. Partial differential equations are used since in each case only one direction is being considered.

$$q = q_x + q_y + q_z = \rho \; dx\, dy\, dz\, C_\rho \frac{\delta T}{\delta t}$$

$$q_x = k \; dy \; dz \left[\left.\frac{\partial T}{\partial x}\right|_1 - \left.\frac{\partial T}{\partial x}\right|_2 \right]$$

$$q_y = k \; dx \; dz \left[\left.\frac{\partial T}{\partial y}\right|_1 - \left.\frac{\partial T}{\partial y}\right|_2 \right]$$

$$q_z = k \; dx \; dy \left[\left.\frac{\partial T}{\partial z}\right|_1 - \left.\frac{\partial T}{\partial z}\right|_2 \right]$$

Combining:

$$\rho C_p \frac{\partial T}{\partial t} = k \left[\frac{\left.\frac{\partial T}{\partial x}\right|_1 - \left.\frac{\partial T}{\partial x}\right|_2}{dx} + \frac{\left.\frac{\partial T}{\partial y}\right|_1 - \left.\frac{\partial T}{\partial y}\right|_2}{dy} \right] + k \left[\frac{\left.\frac{\partial T}{\partial z}\right|_1 - \left.\frac{\partial T}{\partial z}\right|_2}{dz} \right]$$

The difference in the first derivatives divided by dx, dy, or dz is a second derivative, therefore:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] \tag{7.19}$$

Equation (7.19) represents Fourier's second law of heat transfer. The rate of change of temperature with time at any position within a solid conducting heat is proportional to the second derivative of the temperature with respect to distance at that particular point. The ratio $k/(\rho \cdot C_p)$ is α, the thermal diffusivity. The spreadsheet program for k (Fig. 7.1) and C_p (Chapter 3, Fig. 3.1) can be used to estimate the thermal diffusivity of foods from their composition. An Excel program for calculating α determined from food composition is given in Appendix A.11.

7.1.5 Temperature Profile for Unidirectional Heat Transfer Through a Slab

If heat is transferred under steady-state conditions, and A is constant along the distance, x, the temperature gradient, dT/dx will be constant, and integration of Equation (7.1) will result in an expression for temperature as a linear function of x. Substituting the boundary conditions, $T = T_1$ at $x = x_1$, in Equation (7.20):

$$T = -\frac{q/A}{k} \cdot x + C \tag{7.20}$$

Substituting the boundary condition, $T = T_1$ at $x = x_1$, in Equation (7.20):

$$T = \frac{q/A}{k} \cdot (x_1 - x) + T_1 \tag{7.21}$$

Equation (7.21) is the expression for the steady-state temperature profile in a slab where A is constant in the direction of x. The temperature gradient, which is constant in a slab transferring heat at a steady state, is equal to the ratio of the heat flux to the thermal conductivity.

If the temperature at two different points in the solid is known (i.e., $T = T_1$ at $x = x_1$ and $T = T_2$ at $x = x_2$), Equation (7.21) becomes:

$$T_1 - T_2 = -\frac{q/A}{k}(x_1 - x_2)$$

$$\frac{q}{A} = -k \cdot \frac{\Delta T}{\Delta x} \tag{7.22}$$

Substituting Equation (7.22) in Equation (7.21):

$$T = -\frac{\Delta T}{\Delta x} \cdot (x_1 - x) + T_1 \tag{7.23}$$

To keep the convention on the signs, and ensure that proper temperatures are calculated at any position within the solid, $\Delta T = T_2 - T_1$ with increasing subscripts in the direction of heat flow, and $\Delta x = x_2 - x_1$ is positively increasing in the direction of heat flow. Thus, if a temperature drop ΔT across any two points in a solid separated by distance Δx, is known, it will be possible to determine the temperature at any other point within that solid using Equation (7.23). Equations (7.22) and (7.23) also show that when heat is transferred in a steady state in a slab, the product, $k \cong (\Delta T/\Delta x)$, is a constant, and:

$$k\left[\frac{\Delta T}{\Delta x}\right]_1 = k\left[\frac{\Delta T}{\Delta x}\right]_2 = \cdots \tag{7.24}$$

If the solid consists of two or more layers having different thermal conductivity, the heat flux through each layer is constant, therefore the temperature profile will exhibit a change in slope at each junction between layers. A composite slab containing three materials having thermal conductivity k_1, k_2, and k_3, is shown in Fig. 7.5. The temperature profile shows a different temperature gradient within each layer. For a composite slab, Equation (7.24) becomes:

$$k_1\left(\frac{\Delta T_1}{\Delta x_1}\right) = k_2\left(\frac{\Delta T_2}{\Delta x_2}\right) = k_3\left(\frac{\Delta T_3}{\Delta x_3}\right) \tag{7.25}$$

Equation (7.25) can be used to experimentally determine the thermal conductivity of a solid by placing it between two layers of a second solid and determining the temperature at any two points within each

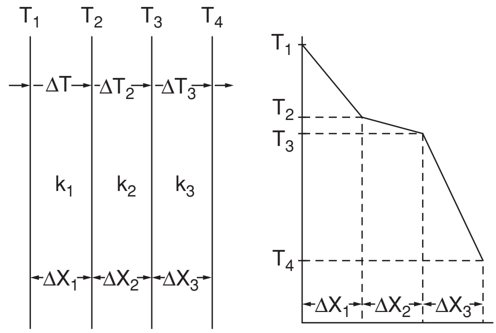

Figure 7.5 Diagram of composite slab and temperature drop at each layer.

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layer of solid after a steady state has been achieved. The existence of a steady state and unidimensional heat flow can be proved by equality of the first and third terms. Two of the terms involving a known thermal conductivity k_1 and the unknown thermal conductivity k_2 can be used to determine k_2.

Example 7.3. Thermocouples embedded at two points within a steel bar, 1 and 2 mm from the surface, indicate temperatures of 100°C and 98°C, respectively. Assuming no heat transfer occurring from the sides, calculate the surface temperature.

Solution:

Because the thermal conductivity is constant, either Equations (7.23) or (7.25) can be used. Following the convention on signs for the temperature and distance differences, $T_2 = 98$, $T_1 = 100$, $x_2 = 2$ mm and $x_1 = 1$ mm. The temperature gradient $\Delta T/\Delta x = (T_2 - T_1)/(x_2 - x_1) = (98 - 100)/0.001(2 - 1) = -2000$. Equation (7.23) becomes:

$$T = -(-2000)(x_1 - x) + T_1$$

At the surface, $x = 0$, and at point $x_1 = 0.001$, $T_1 = 100$. Thus:

$$T = 2000(0.001) + 100 = 102°C$$

Example 7.4. A cylindrical sample of beef 5 cm thick and 3.75 cm in diameter is positioned between two 5-cm-thick acrylic cylinders of exactly the same diameter as the meat sample. The assembly is positioned inside an insulated container such that the bottom of the lower acrylic cylinder contacts a heated surface maintained at 50°C, and the top of the upper cylinder contacts a cool plate maintained at 0°C. Two thermocouples each are embedded in the acrylic cylinders, positioned 0.5 cm and 1.5 cm from the sample-acrylic interface. If the acrylic has a thermal conductivity of 1.5 W/(m · K), and the temperatures recorded at steady state are, respectively, 45° C, 43° C, 15° C, and 13°C, calculate the thermal conductivity of the meat sample.

Solution:

Equation (7.25) will be used. Proceeding along the direction of heat flow, Let $T_1 = 45$, $T_2 = 43$, T_3 = temperature at the meat and bottom acrylic cylinder interface, T_4 = temperature at the meat and top acrylic cylinder interface, $T_5 = 15$, and $T_6 = 13°C$. Let $x_1 = 0$ at the lowermost thermocouple location; $x_2 = 1$; $x_3 = 1.5$; $x_4 = 6.5$; $x_5 = 7$; $x_6 = 8$ cm. Because the temperatures have reached a steady state, the rate of heat transfer across any part of the system are equal therefore the temperature differences indicated by thermocouples within the acrylic should be equal if they are separated the same distances apart. Thus: $(T_2 - T_1)/(x2 - x1) = (T6 - T5)/(x6 - x5) = -2/0.01 = -200$. The heat flux through the system is

$$\frac{q}{A} = -k\left(\frac{\Delta T}{\Delta x}\right) = -1.5(-200) = 300 \text{ W}$$

Using Equation (7.23), the temperatures at the acrylic and meat sample interfaces are

$$T_3 = -(-200)(0.01 - 0.015) + 43 = 42°C$$
$$T_4 = -(-200)(0.07 - 0.065) + 15 = 16°C$$

Using Equation (7.25), with ΔT as the temperature drop across the meat sample $(T_3 - T_4)$, and Δx = the thickness of the meat sample, the thermal conductivity of the meat is

$$k = \frac{q/A}{\Delta T/\Delta x} = \frac{300}{26/0.05} = 0.5769 \text{W}/(\text{m} \cdot \text{K})$$

7.1.6 Conduction Heat Transfer Through Walls of a Cylinder

When heat flows through the walls of a cylinder, the area perpendicular to the direction of heat flow changes with position. A in Equation (7.1) at any position within the wall is the surface area of a cylinder of radius r, and equation 1 may be expressed as:

$$q = -k(2\pi rL)\frac{dT}{dr}$$

When heat transfer occurs in a steady state, q is constant.

$$-\frac{dr}{r} = \frac{2\pi Lk}{q}dT \tag{7.26}$$

After integration and substitution of the boundary conditions: at $r = r_1$, $T = T_1$ and at $r = r_2$, $T = T_2$, the resulting equation for the rate of heat transfer expressed in Equation (7.27) is obtained.

$$q = \frac{T_1 - T_2}{[\ln(r_2/r_1)/2]} \tag{7.27}$$

Equation. (7.27) follows the convention established in Equation (7.24), that q is positive in the direction of decreasing temperature. Increasing subscripts represent positions proceeding farther away from the center of the cylinder.

7.1.7 The Temperature Profile in the Walls of a Cylinder in Steady-State Heat Transfer

At any point r, the temperature T may be obtained from Equation (7.27) by substituting r for r_2 and T for T_2.

$$T = \frac{\ln(r/r_1)q}{2\pi Lk} + T_1$$

Substituting Equation (7.27) for q:

$$T = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_1 - T_2) + T_1 \tag{7.28}$$

or:

$$\frac{(T - T_1)}{\ln(r/r_1)} = \frac{(T_1 - T_2)}{\ln(r_2/r_1)} \tag{7.29}$$

If the wall of the cylinder consists of layers having different thermal conductivities, the temperature profile may be obtained using the same procedure used in deriving Equations (7.28)

$$\frac{(T - T')}{\frac{1}{k_2}\ln\left(\frac{r}{r'}\right)} = \frac{(T_1 - T_2)}{\frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right)} = \frac{q}{2\pi L} \tag{7.30}$$

and (7.29), except that the thermal conductivity of the different layers are used.

In Equation (7.30), the temperatures T_1 and T_2 must transect a layer bounded by r_1 and r_2, which has a uniform thermal conductivity k_1. Similarly, the layer bounded by r and r' where the temperatures are T and T' must also have a uniform thermal conductivity, k_2. Each of the terms in Equation (7.30) is proportional to the rate of heat transfer. Equations (7.30) and (7.25) represent a fundamental relationship in steady-state heat transfer by conduction (i.e., the rate of heat flow through each layer in a multilayered solid is equal).

Example 7.5. A $^3/_4$-in. steel pipe is insulated with 2-cm-thick fiberglass insulation. If the inside wall of the pipe is at 150°C, and the temperature at the outside surface of the insulation is 35°C, (a) calculate the temperature at the interface between the pipe and the insulation and (b) calculate the rate of heat loss for one meter length of pipe.

Solution:

From Table 6.2 on pipe dimensions, the inside diameter of a $^3/_4$-in. steel pipe is 0.02093 m and the outside diameter is 0.02667 m. From Appendix Table A.9, the thermal conductivity of steel is 45 W/(m · K) and that of fiberglass is 0.035 W/(m · K). From the pipe dimensions, $r_1 = 0.01047$ m, $r_2 = 0.01334$, and $r_3 = (0.01334 + 0.020) = 0.03334$ m. $T_1 = 150$.$T_2 =$ the temperature at the pipe-insulation interface. $T_3 = 35$°C. T_1 and T_2 transect the metal wall of the pipe, with k = 45 W/(m · K). From Equation (7.30): $T = T_2$; $T' = T_s$; $r = r_3$ and $r\prime = r_2$; $k_1 = 0.035$ W/(m · K); and $k_2 = 45$ W/(m · K). Substituting in Equation (7.30):

$$\frac{(T_2 - T_s)k_1}{\ln(r_3/r_2)} = \frac{(T_1 - T_2)k_2}{\ln(r_2/r_1)}$$

$$0.038210\ T_2 - 1.33735 = 185.75(150) - 185.75\ T_2$$

$$\frac{(T_2 - 35)(0.035)}{\ln(0.03334/0.01334)} = \frac{(150 - T_2)(45)}{\ln(0.01334/0.01047)}$$

$$T_2(185.75 + 0.03821) = 27,864$$

$$T_2 = 149.97°C$$

7.1.8 Heat Transfer by Convection

This mechanism transfers heat when molecules move from one point to another and exchanges energy with another molecule in the other location. Bulk molecular motion is involved in convection heat transfer. Bulk molecular motion is induced by density changes associated with difference in fluid temperature at different points in the fluid, condensation, or vaporization (natural convection), or when a fluid is forced to flow past a surface by mechanical means (forced convection). Heat transfer by convection is evaluated as the rate of heat exchange at the interface between a fluid and a solid. The rate of heat transfer by convection is proportional to the temperature difference and is expressed as:

$$q = hA(T_m - T_s) = hA\,\Delta T \tag{7.31}$$

where h is the heat transfer coefficient, A is the area of the fluid-solid interface where heat is being transferred, and ΔT, the driving force for heat transfer, is the difference in fluid temperature, T_m, and the solid surface temperature, T_s. Convection heat transfer is often represented as heat transfer through a thin layer of fluid that possesses a temperature gradient, at the fluid-surface interface. The temperature, which is assumed to be uniform at T_m in the fluid bulk, gradually changes through the fluid film until it assumes the solid surface temperature past the film. Thus, the fluid film may be considered as an insulating layer that resists heat flow between the fluid and the solid. The fluid film is actually a boundary layer that has different properties and different velocity from the bulk of the fluid. The magnitude of the heat transfer coefficient varies in an inverse proportion to the thickness of the boundary layer. Conditions that result in a reduction of the thickness of this boundary layer will promote heat transfer by increasing the value of the heat transfer coefficient.

7.1.8.1 Natural Convection

Natural convection depends on gravity and density and viscosity changes associated with temperature differences in the fluid to induce convective currents. The degree of agitation produced by the convective currents depends on the temperature gradient between the fluid and the solid surface. When the ΔT is small, convective currents are not too vigorous, and the process of heat transfer is referred to as free convection. The magnitude of the heat transfer coefficient in free convection is very low, of the order 60 $W/(m^2 \cdot K)$ for air, and 60 to 3000 $W/(m^2 \cdot K)$ for water. When the surface is in contact with a liquid, and the surface temperature exceeds the boiling point of the liquid, bubbles of superheated vapor are produced at the solid-liquid interface. As these bubbles leave the surface, the boundary layer is agitated resulting in very high heat transfer coefficients. This process of heat transfer is called nucleate boiling, and the magnitude of the heat transfer coefficient for water is of the order 5000 to 50,000 $W/(m^2 \cdot K)$. When the ΔT is very high, excessive generation of vapor at the interface produces an insulating layer of vapor that hinders heat transfer. This process of heat transfer is called film boiling, and the heat transfer coefficient is much lower than that in nucleate boiling.

Another form of natural convection is the transfer of heat from condensing vapors. Condensing vapors release a large amount of energy on condensation, therefore, heat transfer coefficients are very high. When the vapors condense as droplets, which eventually coalesce and slide down the surface, the vapor is always in direct contact with a clean surface, and therefore, heat transfer coefficients are very high. This type of heat transfer is called dropwise condensation. Heat transfer coefficients of the order 10,000 $W/(m^2 \cdot K)$ are common in dropwise condensation. When vapors condense as a film of liquid on a surface, the liquid film forms a barrier to heat transfer and heat transfer coefficients are lower. This process of heat transfer is called filmwise condensation, and the magnitude of the heat transfer coefficient may be in the order of 5000 $W/(m^2 \cdot K)$.

7.1.8.2 Forced Convection

In forced convection heat transfer, heat transfer coefficients depend on the velocity of the fluid, its thermophysical properties, and the geometry of the surface. In general, heat transfer coefficients for noncondensing gases are about two orders of magnitude lower than that for liquids. Techniques for calculating heat transfer coefficients are discussed in section “ALocal Heat Transfer Coefficients.”

7.1.9 Heat Transfer by Radiation

Heat transfer by radiation is independent of and additive to that transferred by convection. Electromagnetic waves traveling through space may be intercepted by a suitable surface and absorbed, raising the energy level of the intercepting surface. When the electromagnetic waves are of the frequency of light, the phenomenon is referred to as radiation. All bodies at temperatures higher than absolute zero emit energy in proportion to the fourth power of their temperatures. In a closed system, bodies exchange energy by radiation until their temperatures equalize. Radiation heat transfer, like convection, is a surface phenomenon, therefore the conditions at the surface determine the rate of heat transfer. Thermal radiation includes the spectrum ranging from the high ultraviolet (0.1 μm) through the visible spectrum (0.4 to 0.7 μm) to infrared (0.7 to 100 μm). Surfaces emit thermal radiation in a range of wavelengths, therefore radiation may be expressed as a *spectral intensity*, which is the intensity at each wavelength, or a *total intensity*, which is the integral of the energy emitted over a range of wavelengths. Surfaces also receive energy from the surroundings. The total energy received by a surface is called *irradiation*, which could also be considered as *spectral* or total. The irradiation received by a surface may be absorbed or reflected. The total energy leaving a surface is the sum of the emitted energy by virtue of its temperature and the reflected energy and is called the *radiosity.* The fraction of incident energy absorbed by a surface is called the *absorptivity* (α) and the fraction reflected is called the *reflectivity*(ρ). Some of the energy may be transmitted across the surface, and the fraction of incident energy transmitted is the *transmissivity* (τ). Thus, $\alpha + \rho + \tau = 1$.

7.1.9.1 Types of Surfaces

Surfaces may be classified according to their ability to absorb radiation, as *black* or *gray* bodies. A black body is one that absorbs all incident radiation. An example of a black body is the interior of a hollow sphere that has a small opening to admit radiation. All the energy that enters the small opening is reflected back and forth within the inside of the sphere and, eventually, it will be totally absorbed. Emissivity (ε) is a property that is the fraction of radiation emitted or absorbed by a black body at a given temperature that is actually emitted or absorbed by a surface at the same temperature. Thus black bodies have $\varepsilon = 1$. If a surface absorbs a fraction ($\varepsilon < 1$) of incident radiation equally at all wavelengths , the surface is considered gray. Table 7.1 shows emissivity of some surfaces.

7.1.9.1.1 Kirchoff’s Law. A body at constant temperature is in equilibrium with its surroundings, and the amount of energy absorbed by radiation will be exactly the same as that emitted. Thus, the absorptivity of a surface (α) is exactly the same as the emissivity (ε), and these two properties may be used interchangeably.

Table 7.1 Emissivity of Various Materials

Material	*Temp.('C)*	*Emissivity*
Aluminum, bright	170	0.04
Aluminum paint	100	0.3
Chrome, polished	150	0.06
Iron, hot rolled	20	0.77
Brick, mortar, plaster	20	0.77
Glass	90	0.94
Oil paints, all colors	212	0.92–0.96
Paper	95	0.92
Porcelain	20	0.93
Stainless steel, polished	20	0.24
Wood	45	0.82–0.93
Water	32–212	0.96

Source: Bolz; R. E., and Tuve, G. L. (eds.) 1970 *Handbook of tables for Applied Engineering Science.* CRC Press, Cleveland, Ohio.

7.1.9.1.2 Stephan-Boltzman Law. The energy flux emitted by a black body is directly proportional to the fourth power of the absolute temperature.

$$\frac{q}{A} = \sigma T^4 \tag{7.32}$$

σ is the Stephan-Boltzman constant which has a value of 5.6732×10^{-8} W/($m^2 \cdot K^4$). For gray bodies:

$$q = A\sigma\varepsilon T^4 \tag{7.33}$$

The intensity of energy flux radiating from a black surface is a function of the wavelength, and a wavelength exists where the intensity of radiation is maximum. The energy flux from a black

surface at an absolute temperature T, as a function of the wavelength λ is

$$\frac{q}{A} = \frac{C_1}{\lambda^5} \cdot \frac{1}{\left[e^{C_2/(\lambda T)} - 1\right]} \tag{7.34}$$

C_1 and C_2 are constants. The total flux over the entire spectrum will be:

$$\frac{q}{A} = \int_{-\infty}^{\infty} \left[\frac{C_1\lambda^{-5}}{e^{C_2/(\lambda T)}} - 1\right] d\lambda \tag{7.35}$$

Equation (7.34) can be used to show that there will be a wavelength where the energy flux is maximum. The total energy flux over the whole spectrum is the energy radiated by a body at a temperature, T; thus, Equation (7.35) is equivalent to Equation (7.32).

7.1.9.1.3 Wein's Displacement Law. The wavelength for maximum energy flux from a body shifts with a change in temperature. The product of the wavelength for maximum flux intensity and absolute temperature is a constant. $\lambda_{max} \cdot T = 2.884 \times 10^{-3}$ m $\cdot$ K.

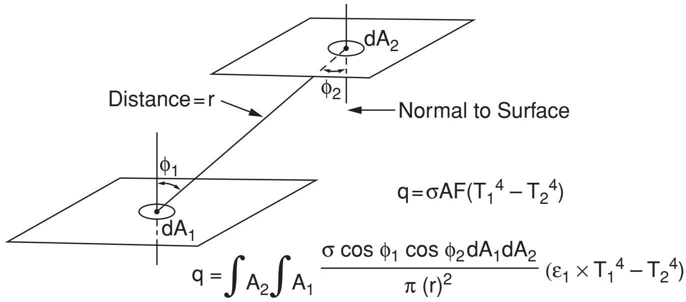

Figure 7.6 Two area elements and representation of the cosine law for heat transfer by radiation.

7.1.9.1.4 Lambert's Law. This is also called the cosine law. The energy flux over a solid angle ω in a direction φ from a normal drawn towards the surface, is a function of cos φ.

$$\frac{dq}{dA\, d\omega} = \frac{\varepsilon\sigma T^4}{\pi}\cos\phi \tag{7.36}$$

Equation (7.36) is the basis for the derivation of view factors used in calculating rate of radiation heat transfer betweem two bodies having the same emissivities. Figure 7.6 shows two area increments transferring heat by radiation and the representation of the cosine law according to Equation (7.36). The solid angle dω with which the area element dA_1 is viewed from dA_2 is

$$d\omega = \frac{\cos\phi_2 dA_2}{r^2}$$

Integrating Equation (7.36) after substituting the expression for the solid angle (dω), will give the rate of heat transfer (q_{1-2}) from area 1 to area 2.

$$q_{1-2} = \varepsilon_1\sigma({T_1}^4 - {T_2}^4)\int_{A_1}\int_{A_2}\frac{\cos\phi_2 dA_2}{\pi r^2}\cdot dA_1\cos\phi_1$$

The double integral times $1/A_1$ is the view factor (F_{1-2}) used to calculate the energy transferred by area 1 and when the multiplier is $1/A_2$ it becomes the view factor (F_{2-1}) for area 2. The equations for the rate of heat transfer become:

$$q_{2-1} = (F_{2-1})A_2\varepsilon\sigma\left({T_1}^4 - {T_2}^4\right) \tag{7.37}$$

$$q_{1-2} = (F_{1-2})A_1\varepsilon\sigma\left({T_1}^4 - {T_2}^4\right) \tag{7.38}$$

Because in a steady state the rate of heat transfer from A_1 to A_2 is the same as that from A_2 to A_1, the product of A_1 and the view factor for A_1 is equal to the product of A_2 and the view factor for A_2. If the two surfaces have different emissivities, the effective view factor ($\overline{F}$) derived by Jacob and Hawkins (1957) is

$$\frac{1}{\overline{\overline{F}}_{2-1}A_2} = \frac{1}{A_2F_{2-1}} + \frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) \tag{7.39}$$

$$\frac{1}{\overline{\overline{F}}_{1-2}A_1} = \frac{1}{A_1F_{1-2}} + \frac{1}{A_1}\left(\frac{1}{\varepsilon_1} - 1\right) + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) \tag{7.40}$$

The rate of heat transfer based on area A_1 or A_2 using the effective view factor is

$$q = A_1\overline{F}_{1-2}\sigma\left(T_1^4 - T_2^4\right) = A_2\overline{F}_{2-1}\sigma\left(T_1^4 - T_2^4\right) \tag{7.41}$$

View factors for some geometries and procedures for applying view factor algebra are available in the literature (e.g., Rohsenow and Hartnett, 1973). View factors for simple geometries are given in the succeeding section on "Radiant Energy Exchange."

7.1.9.2 Effect of Distance Between Objects on Heat Transfer

The total energy intercepted by an area from a point on another area is dependent on the solid angle with which the point views the area. The surface area seen from a point over a solid angle ω may be considered as the base of a cone of distance r from the point and has an area of ωr^2. Because the energy leaving the point toward this viewed area is q, the energy flux is $q/\omega r^2$, indicating that flux is inversely proportional to the square of the distance from the source. Radiant energy flux from a source weakens as a body moves away from a source.

7.1.9.3 Radiant Energy Exchange

View factors for simple geometries:

1. Small object (A_1) surrounded by a large object:

   $$\overline{F}_{1-2} = \varepsilon_1; \quad F_{1-2} = 1$$

2. Large parallel planes with equal areas:

   $$\frac{1}{\overline{F}_{1-2}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \tag{7.42}$$

3. Two parallel disks with centers directly in line (from Rohsenow and Hartnett, 1973): a = diameter of one disk and b = diameter of the other; c = distance between disks. A_1 is the area of the larger disk with diameter b (Fig. 7.7):

   $$\overline{F}_{1-2} = 0.5\left[Z - \sqrt{(Z^2 - 4X^2Y^2)}\right] \tag{7.43}$$

   where $X = a/c$; $Y = c/b$, and $Z = 1 + (1 + X^2)Y^2$.
4. Two parallel long cylinders of equal diameters, b separated by distance 2a (From Roshenow and Hartnett, 1973) (See Figure 7.7):

   $$(\overline{F})_{1-2} = \frac{2}{\pi}\left[\sqrt{(X^2 - 1)} - X + \frac{\pi}{2} - \text{arc cos}\,(1/x)\right] \tag{7.44}$$

   where $X = 1 + a/b$.

A spreadsheet program in Excel for solving the view factors represented by Equations (7.44) is shown in Figure 7.8.

Example 7.6. Glass bottles may be prevented from breaking on filling with hot pasteurized juice when their temperature is close to that of the juice being filled. The bottles are rapidly heated by passing through a chamber that has top, bottom, and side walls heated by natural gas. The glass bottles may be considered as an object completely surrounded by a radiating surface. The glass bottles have

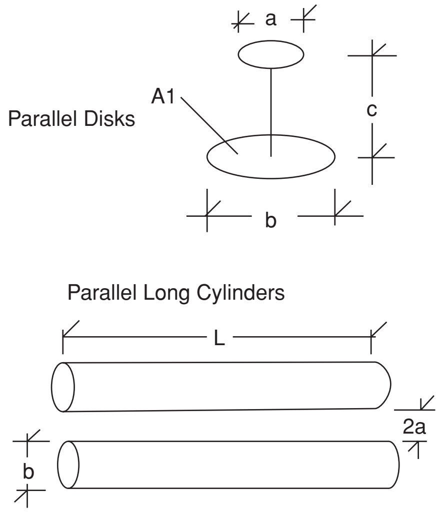

Figure 7.7 Figure of two parallel disks and two parallel long cylinders exchanging energy by radiation.

an emissivity of 0.94, a mass of 155 g each, a specific heat of 1256 J/(kg · K), and a surface area of 0.0219 m^2. If the glass bottles are to be heated from 15.5°C to 51.6°C in 1 minute, calculate the temperature of the walls of the chamber to achieve this average heating rate when the glass is at the midpoint of the temperature range (33.6°C).

Solution:

The required heat transfer rate is $m\ C_p \Delta T$:

$$q = \frac{0.155(1256)(51.6 - 15.5)}{60} = 117.1\,\mathrm{W}$$

Equations (7.39) and (7.41) may be used to calculate $\overline{F}_{1-2}$ and q_{1-2}. Because A_2 in Equation (7.29) is large, the third term on the right of the equation is zero, and the because $F_{1-2} = 1$, $\overline{F}_{1-2} = \varepsilon$.
The heat transfer by radiation may also be calculated using Equation (7.38):

$$F_{1-2} = 1; \quad T_2 = 33.6 + 273 = 303.6\ \mathrm{K}$$

$$q = A\ \varepsilon\ \sigma\left(T_1^4 - T_2^4\right)$$

$$q = 0.0219(0.94)(5.6732e - 0.8)(T^4 - (306.6)^4)$$

$$T^4 = \frac{117.1}{0.0219(0.94)(5.6732e - 0.8)} + (306.6)^4$$

$$T = (1.091034 \times 10^{11})^{0.25} = 574.7\,\mathrm{K}$$

Example 7.7. Cookies traveling on a conveyor inside a continuous baking oven. occupy most of the area on the surface of the conveyor. The top wall of the oven directly above the conveyor has an emissivity of 0.92, and the cookies have an emissivity of 0.8. If the top wall of the oven has a

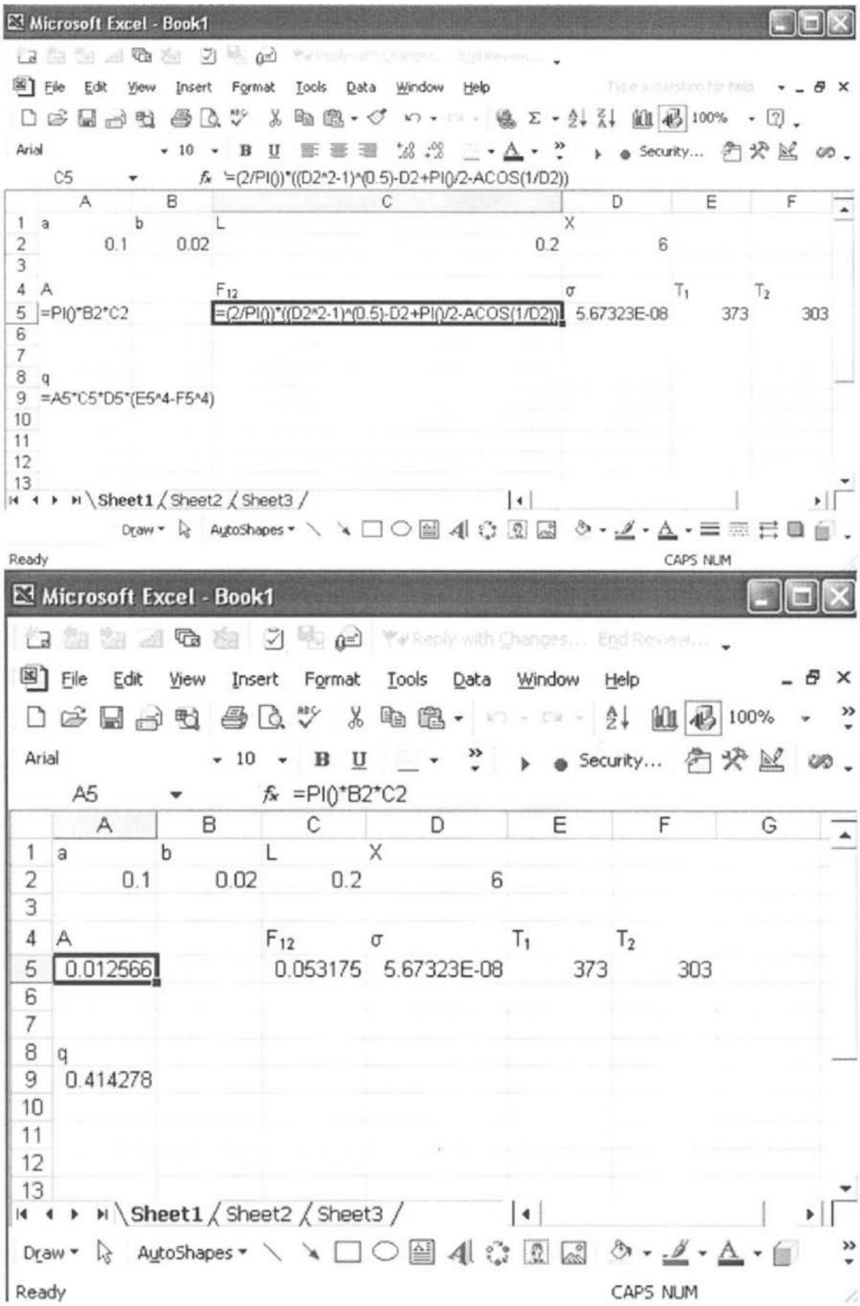

Figure 7.8 Spreadsheet program in excel to solve view factors Equations 7.43 and 7.44.

temperature of 175°C, calculate the average rate of heat transfer by radiation between the cookies per unit area on the side that faces the top wall of the oven when the cookie surface temperature is 70°C.

Solution:

Assume the layer of cookies on the conveyor and the top oven wall constitute a set of long parallel plates. The view factor $\overline{F}_{1-2}$ is calculated using Equation (7.42):

$$\frac{1}{\overline{F}_{1-2}} = \frac{1}{0.92} + \frac{1}{0.80} - 1 = 1.3369$$

$$\overline{F}_{1-2} = \frac{1}{1.3369} = 0.748$$

$$T_1 = 70 + 273 = 343K; T_2 = 175 + 273 = 448K$$

Using Equation (7.41):

$$\frac{q}{A} = 0.748(5.6732 \times 10^{-8})(448^4 - 343^4) = 1122\ W/m^2$$

Example 7.8. The temperature registered by a thermocouple in a gas stream is often the result of steady-state heat transfer by convection from the gas to the thermocouple junction. However, when radiation heat transfer from a surrounding wall is significant, the actual temperature indicated by the thermocouple will be higher than the gas temperature. Consider a superheater for steam that consists of a gas-heated pipe through which steam is flowing. The pipe wall temperature is 500°C and the heat transfer coefficient between the thermocouple junction and the superheated air is 342 W/m^2K. If the thermocouple reads 200°C, what is the actual temperature of the superheated steam. Assume the thermocouple junction is a black body.

Solution:

The thermocouple junction may be considered as an area totally enclosed by another, therefore $F_{1-2} = 1$. Because the temperature of the thermocouple junction is higher than that of steam, energy will be lost by the junction due to convection. To maintain a constant temperature, the heat transfer by radiation will equal that lost by convection and the heat balance becomes:

$$hA(T - T_s) = \sigma\ A\left(T_w{}^4 - T^4\right)$$

Solving for T_s:

$$T_s = (1/h)\left[hT - \sigma\ \left(T_w{}^4 - T^4\right)\right]$$

T_w and T in the expression for radiation heat transfer is in absolute temperature of 773 and 473 K, respectively, therefore for consistency T and T_s in the expression for convection heat transfer should also be expressed in Kelvin. Solving for T_s in Kelvin:

$$T_s = (1/342)\left[342A\ 473 - 5.6732 \times 10^{-8}\ \left\{(773)^4 - (473)^4\right\}\right] = 422\ K \text{ or } 149°C$$

Note that the actual steam temperature T_s will approach the indicated temperature T as h approaches infinity or if T_w approaches T.

Example 7.9. A vacuum belt dryer consists of a belt traveling over a heated plate that conveys the product and a heated plate positioned over the belt the whole length of the belt. Heat transfer from the heated top plate to the product on the conveyor is by radiation and the view factor may be considered as that for long parallel plates. The top plate is sand blasted stainless steel with $\varepsilon = 0.52$. Apple slices $\varepsilon = 0.85$ are being dried and the desired product temperature during drying equals the boiling temperature of water at an absolute pressure of 7 mm Hg, the pressure inside the vacuum dryer. If the surface area of sliced apples at 86% water is 0.731 cm^2/g, calculate the rate of evaporation of water from the apple surface due to heat transfer by radiation, expressed in g water evaporated/(h Ag dry apple solids) if the top plate temperature is maintained at 80°C.

Solution:

At 7 mm Hg absolute pressure, the pressure in Pascals is

$$P = 0.7 \text{ cm Hg} \times 1333.33 \text{ Pa/cm Hg} = 933.33 \text{ Pa}$$

From Appendix table A.4, $T = 5 + [2.5/(1.0365 - 0.8724)](0.93333 - 0.8724) = 5.9°C$
Using Equation (7.42): $\overline{F}_{1-2} = 1/[1/0.52 + 1/.85 - 1] = 0.4763$
Using Equation (7.41): $q/A_1 = 0.4763(5.6732 \times 10^{-8})[(353)^4 - (278.9)^4] = 256.07 W/m^2$
From Appendix Table A.4, the latent heat of evaporation of water at .9333 kPa is

$$h_{fg} = 2.4897 - [(2.4897 - 2.4839)/(1.0365 - 0.8724)](0.9333 - 0.8724) = 2.4918 \text{ MJ/kg}$$

Drying rate:

$$\begin{aligned} dW/dt &= [256.07 \text{ J/(s A m}^2)][\text{kg}/2,491,800 \text{ J}][1000\text{g/kg}][3600 \text{ s/h}] \\ &= 369.95 \text{ g water/(m}^2 \text{ A h)} \end{aligned}$$

The apple surface area in m^2/g dry matter:

$$= \left[0.731 \text{ cm}^2/(1 - 0.86) \text{ g dry matter}\right]\left[1 \text{ m}^2/10000 \text{ cm}^2\right] = 0.000522$$

$$dW/dt = [369.95 \text{ g water/(m}^2 \text{ A h)}][0.000522 \text{ m}^2\text{/g dry matter}]$$

= 0.193 g water/(h A g dry apple solids) evaporation from the radiant heat transfer alone.

7.1.10 Microwave and Dielectric Heating

Microwaves like light are also electromagnetic vibration. Heat transfer is dependent on the degree of excitability of molecules in the absorbing medium and the frequency of the field to which the medium is exposed. Dielectric heating is the term used when relatively low frequencies are used and the material is placed between two electrodes to which an electric current is passed. Frequencies from 60 Hz to 100 MHz may be used for dielectric heating. Microwave heating refers to the use of electromagnetic waves of very high frequency making it possible to transmit the energy through space. The most common frequencies used for microwave heating are 2450 MHz and 915 MHz. Domestic microwave ovens operate at 2450 MHz. The equations that govern heat transfer by microwave and dielectric systems are the same.

7.1.10.1 Energy Absorption by Foods in a Microwave Field

The energy absorbed by a body is

$$\frac{q}{V} = 0.556(10^{-12})\, f\, E^2 e \tan(\delta) \tag{7.45}$$

Table 7.2 Dielectric Properties of Food and Other Materials.

Material	*Temperature °C*	*e″*	*tan(δ)*
Beef (raw)	−15	5.0	0.15
Beef (raw)	25	40	0.30
Beef (roast)	23	28	0.20
Peas (boiled)	−15	2.5	0.20
Peas (boiled)	23	9.0	0.50
Pork (raw)	−15	6.8	1.20
Pork (roast)	35	23.0	2.40
Potatoes (boiled)	−15	4.5	0.20
Potatoes (boiled)	23	38.0	0.30
Spinach (boiled)	−15	13.0	0.50
Spinach (boiled)	23	34.0	0.80
Suet	25	2.50	0.07
Porridge	−15	5.0	0.30
Porridge	23	47.0	0.41
Pyrex	25	4.80	0.0054
Water	1.5	80.5	0.31
water	25	76.7	0.15
0.1 M NaCl	25	75.5	0.24

Sources: Copson, 1971. *Microwave Heating. AVI* Publishing Co. Westport, Conn.; Schmidt, W. 1960. Phillips *Tech. Rev* 3.89.

where q/V = energy absorbed, W/cm^3; f = frequency, Hz; e = dielectric constant, an index of the rate at which energy penetrates a solid; dimensionless tan(δ) = dielectric loss factor, an index of the extent to which energy entering the solid is converted to heat; dimensionless E = field strength in volts/cm^2. e and tan(δ) are properties of the material and are functions of composition and temperature. f and E are set by the type of microwave generator used. Table 7.2 shows the dielectric constant and the dielectric loss factor for foods, food components, and some packaging materials. Metal containers are opaque to microwave (i.e., microwaves are reflected from the surface therefore none passes across to food contained inside). However, an electrically conductive metal finite electrical resistance will heat up in the same manner as an electrical resistance wire will heat up when an electric current is passed through it. Similarly, electrically conductive wires will heat very rapidly in a microwave field. A continuous metal sheet with very low electrical resistance and will not heat up in a microwave field. However, a discontinuous metal sheet such as metallized plastic contains many small areas of metal that presents a large resistance to electrical current flow, therefore intense heating occurs in these materials. Such materials called absorbers or intensifiers are used in microwavable packages of frozen breaded fried foods or pizza to ensure a crispy crust when heated in a microwave. Glass and plastic are practically transparent to microwaves, i.e. they transmit microwaves and very little energy is absorbed.

The frequency of microwave power generated by a microwave generator is declared on the name plate of the unit. The power output is also supplied by the manufacturer for each unit. The coupling efficiency of a microwave unit is expressed as the ratio of power actually supplied to the unit and the actual power absorbed by the material heated. When the quantity of material being heated is large, the power generated by the system limits the power absorbed, rather than the value predicted by Equation

(7.45).The time it takes to heat a large quantity of material can be used to determine the microwave power output of a unit.

If a maximum q/v is determined by varying the quantity of food heated until further reduction results in no further increase in q/v, this value will be the limiting power absorption and will be dependent on the dielectric loss properties of the material according to Equation (7.45). If the dielectric constant and loss tangent of the material are known, it will be possible to determine the electromagnetic field strength which exists, and differences in heating rates of different components in the food mixture can be predicted using Equation (7.45).

7.1.10.2 Relative Heating Rates of Food Components

When the power output of a microwave unit limits the rate of energy absorption by the food, components having differenct dielectric properties will have different heating rates. Using subscripts 1 and 2 to represent component 1 and 2, e″ to represent the product of e and tan(δ), and C to represent the constant, Equation (7.45) becomes:

$$q_1 = \frac{m_1}{\rho_1} C f E^2 e_1''$$

$$q_2 = \frac{m_2}{\rho_2} C f E^2 e_2''$$

Because $P = q1 + q2$; and $q = mC_p\, dT/dt$:

$$C f E^2 = \frac{P}{(m_1/\rho_1)e_1'' + (m_2/\rho_2)e_2''}$$

$$\frac{dT_1}{dt} = \frac{\rho_2 e''_1 P}{C_{p1}(\rho_2 e''_1 m_1 + \rho_1 e''_2 m_2)} \tag{7.46}$$

$$\frac{dT_2}{dt} = \frac{\rho_1 e''_2 P}{C_{p2}(\rho_2 e''_1 m_1 + \rho_1 e''_2 m_2)} \tag{7.47}$$

The relative rate of heating is

$$\frac{dT_1}{dT_2} = \frac{\rho_2 e''_1 C_{p2}}{\rho_1 e''_2 C_{p1}} \tag{7.48}$$

Similar expressions may be derived for more than two components.

Example 7.10. The dielectric constant of beef at 23°C and 2450 MHz is 28 and the loss tangent is 0.2. The density is 1004 kg/m^3 and the specific heat is 3250 J/(kg · K). Potato at 23°C and 2450 MHz has a dielectric constant of 38 and a loss tangent of 0.3. The density is 1010 kg/m^3 and the specific heat is 3720 J/(kg · K).

(a) A microwave oven has a rated output of 600 W. When 0.25 kg of potatoes were placed inside the oven, the temperature rise after 1 minute of heating was 38.5°C. When 60 g of potato was heated in the oven, a temperature rise of 40°C was observed after 20 s. Calculate the average power output of the oven, and the mass of potatoes that must be present such that power output of the oven is limiting the rate of power absorption rather than the capacity of the material to absorb the microwave energy.

(b) If potatoes and beef are heated simultaneously, what would be the relative rate of heating?

Solution:

(a) Assume that 0.25 kg mass of product is sufficient to make microwave power availability the rate limiting factor for microwave absorption.

$$P = 0.25\ \text{kg}\left[\frac{3720\ \text{J}}{\text{kg}\cdot\text{K}}\right]\left[\frac{38.5\ \text{K}}{1\ \text{min}}\right]\left[\frac{1\ \text{min}}{60\ \text{s}}\right] = 596.75\ \text{W}$$

When a small amount of material is heated:

$$P = 0.06\ \text{kg}\left[\frac{3720\ \text{J}}{\text{kg}\cdot\text{K}}\right](40\ \text{K})\left[\frac{1}{20\ \text{s}}\right] = 446.4\ \text{W}$$

The amount absorbed with the small mass in the oven is much smaller than when a larger mass was present; therefore it may be assumed that power absorption by the material limits the rate of heating.

$$\frac{q}{V} = \left[\frac{446.4\ \text{W}}{0.06\ \text{kg}\frac{1\text{m}^3}{1010\,\text{kg}}}\right]\cdot\frac{1\ \text{m}^3}{10^{-6}\ \text{cm}^3} = 7.5144\ \text{W/cm}^3$$

Because with a small load in the oven, the power absorption is only 7.5177 W/cm^3, this may be assumed to be $(q/V)_{\text{lim}}$, the maximum rate of power absorption by the material. If the power output of the oven as calculated above is 596.75 W, the mass present where the rate of power absorption equals the power output of the oven is

$$\left.\frac{q}{V}\right|_{\text{lim}} = \frac{P(\rho)10^{-6}}{m}$$

$$m = \frac{P(\rho)10^{-6}}{(q/V)_{\text{lim}}} = \frac{596.75(1010\times 10^{-6})}{7.5177} = 0.08\ \text{kg}$$

Thus, any mass greater than 0.08 kg will heat inside this microwave oven at the rate determined by the power output of the oven.

(b) Using Equation (7.48): Let subscript 1 refer to beef and subscript 2 refer to the potato.

$$\frac{dT_1}{dT_2} = \frac{1010(28)(0.2)(3720)}{1004(38)(0.3)(3250)} = 0.565$$

Thus, the beef will be heating slower than the potatoes.

7.2 TEMPERATURE MEASURING DEVICES

Temperature is defined as the degree of thermal agitation of molecules. Changes in molecular motion of a gas or liquid will change the volume or pressure of that fluid, and a solid will undergo a dimensional change such as expansion or contraction. Certain metals will lose electrons when molecules are thermally excited and when paired with another metal whose molecules could receive these displaced electrons, an electromotive force will be generated. These responses of materials to changes in temperature are utilized in the design of thermometers, electronic and mechanical temperature measuring devices. The calibration of all temperature measuring devices is based on conditions of use. Some electronic instruments have ambient temperature compensation integrated in the circuitry, and others do not. Similarly, fluid filled thermometers are calibrated for certain immersion depths, therefore partial immersion of a thermometer calibrated for total immersion will lead to errors in measurement. Thus, conditions of use that correspond to the calibration of the instrument must be known, for accurate measurements. For other conditions, the instrument will need recalibration. Thermometers filled with fluids other than mercury may have to be recalibrated after a certain time of use to ensure that breakdown of the fluid, which might change its thermal expansion characteristics, has not occurred. In the food industry, a common method of calibration involves connecting the thermometers to a manifold which contains saturated steam. The pressure of the saturated steam can then be used to determine the temperature from steam tables, and compared to the thermometer readings. Recalibration of fluid filled thermometers is normally not done. Thermometers used in critical control points in processes must be replaced when they lose accuracy. Electronic temperature measuring devices must be recalibrated frequently against a fluid-filled thermometer.

The temperature indicated by a measuring device represents the temperature of the measuring element itself, rather than the temperature of the medium in contact with the element. The accuracy of the measurement would depend on how heat is transferred to the measuring element. The temperature registered by the instrument will be that of the measuring element after heat exchange approaches equilibrium.

Thermometers will not detect oscillating temperatures if the mass of the temperature measuring element is such that the lag time for heat transfer equilibration is large in relation to the period of the oscillation. Response time of temperature measuring devices is directly proportional to the mass of the measuring element; the smaller the mass the more accurate the reading.

Measurement of the surface temperature of a solid in contact with either a liquid or a gas is not very accurate if the measuring element is simply laid on the solid surface. Depending upon the thickness of the measuring element, the temperature indication will be intermediate between that of the surface and the fluid temperature at the interface. Accurate solid surface temperatures can only be determined by embedding two thermocouples a known distance from the surface and extrapolating the temperature readings towards the surface to obtain the true surface temperature.

Measurements of temperatures of gases can be influenced by radiation. As shown in the previous section on Radiation Heat Exchange, an unshielded thermometer or thermocouple in a gas stream surrounded by surfaces of a different temperature the gas will read a temperature different from the true gas temperature because of radiation. Shielding of measuring elements is necessary for accurate measurements of gas temperatures.

7.2.1 Liquid-in-Glass Thermometers

Liquid-in-glass thermometers most commonly use mercury for general use, and mineral spirits, ethanol, or toluene for low temperature use. When properly calibrated, any temperature stable liquid may be used, and non–mercury-filled thermometers are now available for use as temperature indicators in food processing facilities.

7.2.2 Fluid-Filled Thermometers

This temperature measuring device consists of a metal bulb with a long capillary tube attached to it. The end of the capillary is attached to a device that causes a definite movement with pressure transmitted to it from the capillary. The capillary and bulb is filled with fluid that changes in pressure with changes in temperature. Capillary length affects the calibration and so does the ambient temperature surrounding the capillary. Thus, temperature measuring devices of this type must be calibrated in the field. Figure 7.9B shows a typical fluid-filled thermometer with a spiral at the end of the capillary. The movement of the spiral is transmitted mechanically through a rack and pinion arrangement or a lever to a needle which moves to indicate the temperature on a dial. Some stainless steel cased dial type thermometers are of this type. Proper performance of these instruments requires total immersion of the bulb.

7.2.3 Bimetallic Strip Thermometers

When two metals having dissimilar thermal expansion characteristics are joined, a change in temperature will cause a change in shape. For example, thermostats for domestic space cooling and heating utilize a thin strip of metal fixed at both ends to a thicker strip of another metal. An increase in temperature will make the thinner metal, which has a higher thermal coefficient of expansion, to bend and the change in position may open or close properly positioned electrical contacts, which then activates heating or cooling systems. In some configurations, the bimetallic strip may be wound in a helix or a spiral. The strip is fixed at one end, and the free end would move with changes in temperature to position an indicator needle on a temperature scale. This type of thermometer requires total immersion of the bimetallic strip for proper performance.

7.2.4 Resistance Temperature Devices (RTDs)

The principle of RTDs is based on the change in resistance of a material with changes in temperature. Figure 7.9a shows an RTD circuit. The resistance strip may be an insulated coil of resistance wire encased in a metal tube (resistance bulb) or a strip of metal such as latinum. Other materials such as semiconductors exhibit large drop in resistance with increasing temperatures. The whole instrument consists of the resistance or semiconductor element for sensing the temperature, electronic circuitry for providing the excitation voltage, and a measuring circuit for the change in resistance. Response is usually measured as a change in voltage or current in the circuit. RTDs vary in size and shape depending upon use. Total immersion of the sensing element is necessary for accurate results.

7.2.5 Thermocouples

A thermocouple is a system of two separate dissimilar metals with the ends fused together forming two junctions. When the two junctions are at different temperatures, an electromotive force is generated. This electromotive force is proportional to the difference in temperature between the two junctions. A thermocouple circuit is shown in Fig. 7.10. One of the junctions, the reference junction, is immersed in a constant temperature bath which is usually ice water. Electromotive

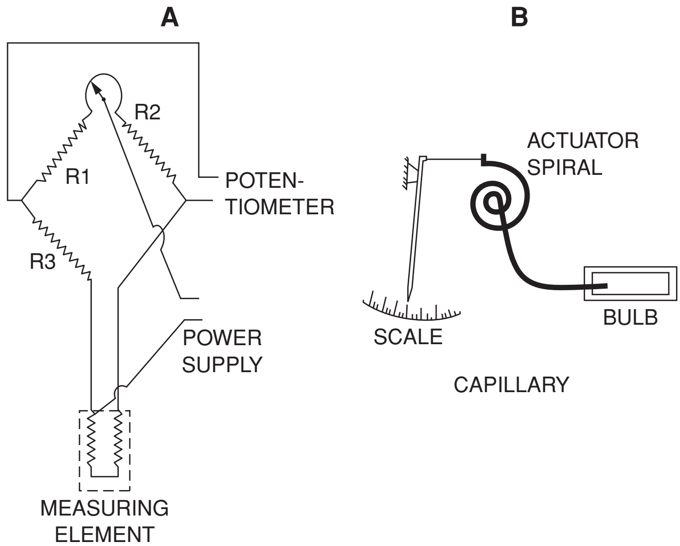

Figure 7.9 Schematic diagram of temperature-measuring devices. (A) RTD circuit and (B) fluid-filled thermometer.

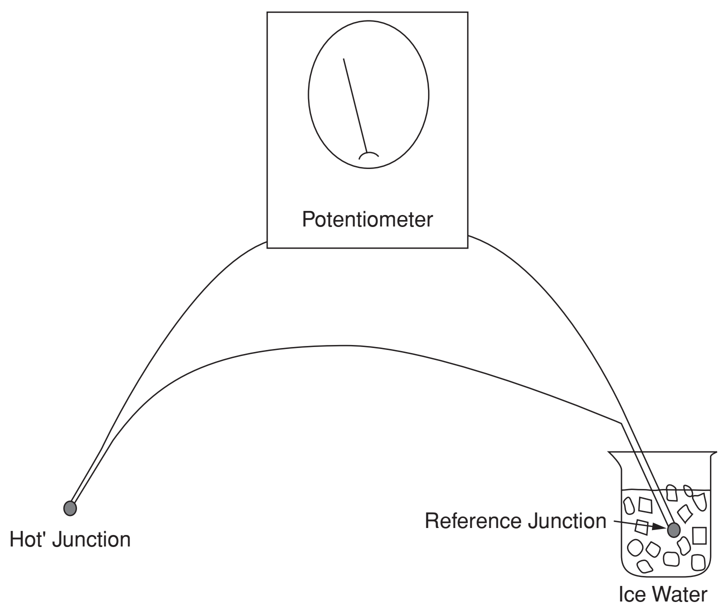

Figure 7.10 Thermocouple circuit showing ice water as the reference.

force values at various temperatures with ice water as the reference are tabulated and available in handbooks.

7.2.6 Radiation Pyrometers

This type of temperature sensing device focuses the energy received from a surface through a lens on a device such as a photovoltaic cell or a thermopile. This energy stimulates the production of an electromotive force which can then be measured. Since the amount of energy received per unit area by a receiving surface from a source, varies inversely as the distance, these types of measuring devices have distance compensation features. Distance compensation is achieved by focusing the energy received on the lens of the instrument into a smaller area having an energy sensitive surface. Radiation pyrometers are useful for sensing very high temperatures such as in furnaces, flames, and red hot or molten metals. Some, which have been calibrated for operation at near ambient temperatures, are very useful for non-contact monitoring of surface temperatures of processing equipment.

7.2.7 Accurate Temperature Measurements

Fluid in glass thermometers: The most common of these is the mercury in glass thermometer, but there are other fluids used. The temperature indicated is based on thermal expansion of the fluid in the thermometer. The temperature markings on the stem are constructed based on the volume of the bulb, the diameter of the glass capillary into which the fluid emerges from the bulb, and the coefficient of thermal expansion of the fluid. The temperature scale in the stem is marked based on a specified *minimum immersion depth*. A *total immersion* thermometer must have the bulb and the whole stem immersed in the fluid to be measured.

Partial immersion thermometers have the minimum immersion depth marked on the stem below the temperature scale. The minimum immersion depth for partial immersion thermometers is usually 76 mm from the tip of the bulb. Errors are introduced when the immersion depth is below the minimum. The magnitude of the error will depend on the length of the exposed mercury column and the ambient temperature around the exposed mercury column.

Thermocouples: Errors in temperature measurement with thermocouples come from improper contact between thermocouple wires at the measuring junction, conduction of heat along the thermocouple wires, which would make the temperature at the measuring juction different from the medium measured; interference with convection heat transfer at the measuring junction; radiation heat transfer between the measuring junction and the surroundings; resistance to current flow by the thermocouple wire, inaccurate reference junction compensation, and accuracy of the instrument that reads the electromotive force developed within the thermocouple circuit. One of the most common problems in temperature measurement is in the selection of the size of thermocouple wire. Although a very thin wire thermocouple eliminates the effect of heat conduction, a long thermocouple lead with thin wires would present a high resistance in the circuit resulting in a significant drop in the measured electromotive force. Another common problem is the joining of the thermocouple wires at the measuring junction. The two wires must have good electrical contact, and the joint must contribute minimum electrical resistance in the thermocouple circuit. With thin wires, the bare ends are twisted at least three turns before soldering or brazing, while with thicker wires, the two ends may simply be butted together and brazed or welded.

7.3 STEADY-STATE HEAT TRANSFER

7.3.1 The Concept of Resistance to Heat Transfer

Equation (7.25), derived from Fourier's first law, shows that in a steady-state system the quantity of heat passing through any part of the system must equal that passing through any other part and equal the total passing through the system. The problem of heat transfer through multiple layers can be analyzed as a problem involving a series of resistance to heat transfer. The transfer of heat can be considered as analogous to the transfer of electrical energy through a conductor. ΔT is the driving force equivalent to the voltage E in electrical circuits. The heat flux q is equivalent to the current, I.

Ohm's law for electrical circuits is

$$I = \frac{E}{R} \tag{7.49}$$

For heat transfer through a slab:

$$\frac{q}{A} = \frac{\Delta T}{[\Delta X/k]} = \frac{\Delta T}{R} \tag{7.50}$$

Thus, in comparing Equations (7.48) and (7.49), R is equivalent to $\Delta X/k$. The resistance to heat transfer is $\Delta X/k$ and is the "R" rating used in the insulation industry to rate the effectiveness of insulating materials. For multilayered materials in the geometry of a slab where A is constant in the direction of increasing x, the overall resistance to heat transfer is the sum of the individual resistance in series, and:

$$R = R1 + R2 + R3 + \ldots\ldots Rn$$

Thus:

$$R = \frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \ldots\ldots \frac{\Delta x_n}{k_n}$$

and:

$$\frac{q}{A} = \frac{\Delta T}{[\Delta x_1/k_1 + \Delta x_2/k_2 \ldots\ldots\ldots \Delta x_n/k_n]} \tag{7.51}$$

Equation (7.50) can also be written as:

$$\frac{q}{A} = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} = \frac{\Delta T_3}{R_3} = \ldots\ldots \frac{\Delta T_n}{R_n} = \frac{\Delta T}{R} \tag{7.52}$$

Equation (7.52) is similar to Equation (7.25) previously derived using Fourier's first law.

For heat transfer through a cylinder, the heat transfer rate in Equation (7.27) results in an expression for the heat transfer resistance as:

$$R = \frac{\ln(r_2/r_1)}{2\pi LK} \tag{7.53}$$

For resistances in series:

$$q = \frac{\Delta T}{R}$$

$$= \frac{\Delta T}{[\ln(r_2/r_1)/2\pi Lk_1 + \ln(r_3/r_2)/2\pi Lk_2 + \ldots \ln(r_{n+1}/r_n)/2\pi Lk_n]} \tag{7.54}$$

and:

$$\frac{\Delta T_1}{[\ln(r_2/r_1)2\pi L k_1]} = \frac{\Delta T_2}{[\ln(r_3/r_2)/2\pi L k_2]} = \ldots \frac{\Delta T_n}{[\ln(r_{n+1}/r_n)/2\pi L k_n]} \tag{7.55}$$

For convection heat transfer:

$$q = h\ A\ \Delta T = \frac{\Delta T}{(1/hA)}$$

$$R = \frac{1}{hA} \tag{7.56}$$

7.3.2 Combined Convection and Conduction: The Overall Heat Transfer Coefficient

Most problems encountered in practice involve heat transfer by combined convection and conduction. Usually, the temperatures of fluids on both sides of a solid are known and the rate of heat transfer across the solid is to be determined. Heat transfer involves convective heat transfer between a fluid on one surface, conductive heat transfer through the solid and convective heat transfer again at the opposite surface to the other fluid. The rate of heat transfer may be expressed in terms of U, the overall heat transfer coefficient, or in terms of R, an overall resistance.

Consider a series of resistances involving n layers of solids and n fluid to surface interfaces. The thermal conductivity of the solids are $k_1 \ldots k_2 \ldots k_3 \ldots k_n$ and the heat transfer coefficients are $h_1 \ldots h_2 \ldots h_n$ with subscript n increasing along the direction of heat flow.

$$q = UA\Delta T = \frac{\Delta T}{R} \tag{7.57}$$

For a slab:
Because A is the same across the thickness of a slab:

$$R = \frac{1}{UA} \tag{7.58}$$

$$R = \sum \left[\frac{1}{h_n A}\right] + \sum \left[\frac{X_n}{k_n A}\right] = \frac{1}{UA}$$

$$\frac{1}{U} = \frac{1}{h_1} + \frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \ldots\ldots\ldots \frac{\Delta x_n}{k_n} + \ldots\ldots\ldots \frac{1}{h_n} \tag{7.59}$$

or:

$$U = \frac{1}{\left[\sum(1/h_n) + \sum(x_n/k_n)\right]} \tag{7.60}$$

For a cylinder:

$$R = \sum \left[\frac{\ln(r_{n+1}/r_n)}{2\pi L k_n}\right] + \sum \left[\frac{1}{h_n A_n}\right] = \frac{1}{UA}$$

If the A used as a multiplier for U is the outside area, then $U = U_o$ the overall heat transfer coefficient based on the outside area. U_i = overall heat transfer coefficient based on the inside area. Using h_i = inside heat transfer coefficient and h_o = outside heat transfer coefficient; r_i and r_o are inside and outside radius of the cylinder, respectively.

$$(2\pi r_o L)U_o = \frac{1}{(1/2\pi L)\sum [\ln(r_{n+1}/r_n)/k_n] + (1/2\pi L)[1/h_o r_o + 1/h_{ir_i}]}$$

or:

$$U_o = \frac{1}{r_o \sum[\ln(r_n/r_{n-1})/k_n] + [r_o/r_i h_i] + [1/h_o]}$$

$$\frac{1}{U_o} = \frac{r_o}{r_i h_i} + \frac{r_o \ln(r_2/r_1)}{k_1} + \frac{r_o \ln(r_3/r_2)}{k_2} + \ldots\ldots \frac{r_o \ln(r_n/r_{n-1})}{k_n} + \frac{1}{h_o} \tag{7.61}$$

$$U_i = \frac{1}{r_i \sum[\ln(r_n/r_{n-1})/k_n] + [r_i/r_o h_o] + [1/h_i]}$$

or:

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i \ln(r_2/r_1)}{k_1} + \frac{r_i \ln(r_3/r_2)}{k_2} + \ldots\ldots \frac{r_i \ln(r_n/r_{n-1})}{k_n} + \frac{r_i}{r_o h_o} \tag{7.62}$$

Example 7.11. Calculate the rate of heat transfer across a glass pane that consists of two 1.6-mm-thick glass separated by 0.8-mm layer of air. The heat transfer coefficient on one side that is at 21°C is 2.84 W/(m$^2\cdot$ K) and on the opposite side that is at −15°C is 11.4 W/(m$^2\cdot$ K). The thermal conductivity of glass is 0.52 W/(m · K) and that of air is 0.031 W/(m · K).

Solution:

When stagnant air is trapped between two layers of glass, convective heat transfer is minimal and the stagnant air layer will transfer heat by conduction. There are five resistances to heat transfer. R_1 is the convective resistance at one surface exposed to air, R_2 is the conductive resistance of one 1.6-mm-thick layer of glass, R_3 is the conductive resistance of the air layer between the glass, R_4 is the conductive resistance of the second 1.6 mm thick layer of glass, and R_5 is the convective resistance of the opposite surface exposed to air. Using Equation (7.9):

$$\begin{aligned}
\frac{1}{U} &= \frac{1}{h_1} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \frac{1}{h_2} \\
&= \frac{1}{2.84} + \frac{1.6\times 10^{-3}}{0.52} + \frac{0.8\times 10^{-3}}{0.031} + \frac{1.6\times 10^{-3}}{0.52} + \frac{1}{11.4} \\
&= 0.352 + 0.0031 + 0.0258 + 0.0031 + 0.0877 = 0.4718 \\
U &= 2.12\frac{W}{m^2\cdot K}
\end{aligned}$$

$$\frac{q}{A} + U\Delta T = 2.12(21 - (-15)) = 76.32\frac{W}{m^2}$$

Example 7.12. (a) Calculate the overall heat transfer coefficient for a 1-in. (nominal) 16-gauge heat exchanger tube when the heat transfer coefficient is 568 W/(m^2 K) inside and 5678 W/(m$^2\cdot$ K) on the outside. The tube wall has a thermal conductivity of 55.6 W/(m · K). The tube has an inside diameter of 2.21 cm and a 1.65 mm wall thickness. (b) If the temperature of the fluid inside the tube is 80°C and 120°C on the outside, what is the inside wall temperature?

Solution:

$$r_i = 2.21/2 = 1.105 \text{ cm}$$
$$r_o = 1.105 + 0.1651 = 1.2701 \text{ cm}$$

(a) Using Equation. (7.61):

$$\frac{1}{U_O} = \frac{1}{5678} + \frac{1.2701 \times 10^2 \ln(1.2701/1.105)}{55.6} + \frac{1.2701 \times 10^{-2}}{(1.105 \times 10^{-2})(568)}$$
$$= 1.76 \times 10^{-4} + 0.318 \times 10^{-4} + 20.236 \times 10^{-4}$$
$$= 22.315 \times 10^{-4}; \quad U_o = 448 \text{W/m}^2 \cdot \text{K}$$

(b) Temperature on the inside wall:

q(overall) = q(across inside convective heat transfer coefficient).

Let: T_f = temperature of fluid inside tube and T_w = temperature of the wall.

$$U_o A_o \, \Delta T = h_i \, A_i \, (T_w - T_f)$$
$$448(2\pi r_o L)(120 - 80) = 568(2\pi r_i L)(T_w - 80)$$
$$(T_w - 80) = \frac{448(r_o)(40)}{568(r_i)}$$
$$= 80 + \frac{448(1.2701)(40)}{568(1.105)}$$
$$= 80 + 36.3 = 116.3°\text{C}$$

7.4 HEAT EXCHANGE EQUIPMENT

Heat exchangers are equipment for transferring heat from one fluid to another. In its simplest form a heat exchanger could take the form of a copper tube exposed to air, where fluid flowing inside the tube is cooled by transferring heat to ambient air. When used for transferring large quantities of heat per unit time, heat exchangers take several forms to provide for efficient utilization of the heat contents of both fluids exchanging heat, and to allow for compactness of the equipment. The simplest heat exchanger to make is one of tubular design. Heat exchangers commonly used in the food industry include the following.

Swept surface heat exchangers: The food product passes through an inner cylinder and the heating or cooling medium passes through the annular space between the inner cylinder and an outer cylinder or jacket. A rotating blade running the whole length of the cylinder continuously agitates the food as it passes through the heat exchanger and at the same time, continuously scrapes the walls through which heat is being transferred (the heat transfer surface). This type of heat exchanger can be used to heat, cool or provide heat to concentrate viscous food products. Figure 7.11 shows a swept surface heat exchanger.

Double pipe heat exchanger: This heat exchanger consists of one pipe inside another. The walls of the inner pipe forms the heat transfer surface. This type of heat exchanger is usually built and

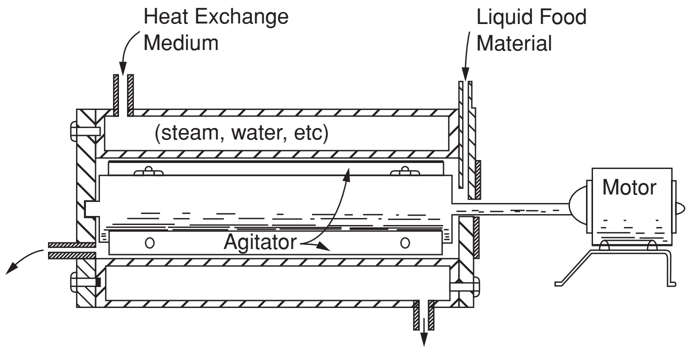

Figure 7.11 Diagram of a swept surface heat exchanger.

installed in the field. A major disadvantage is the relatively large space it occupies for the quantity of heat exchanged, compared to other types of heat exchangers. Figure 7.12 shows a double pipe heat exchanger.

Shell and tube heat exchanger: This heat exchanger consists of a bundle of tubes enclosed by a shell. The type of head arrangement allows for one tube pass or multiple passes for the product. In a one-pass arrangement, the product enters at one end and leaves at the opposite end. In a multipass arrangement, the product may travel back and forth through different tubes with each pass before finally leaving the heat exchanger. The heat exchange medium on the outside of the tubes are usually distributed using a system of baffles. Figure 7.13 shows two types of shell-and-tube heat exchanges.

Plate heat exchanger: This type of heat exchanger was developed for the dairy industry. It consists of a series of plates clamped together on a frame. Channels are formed between each plate. The product and heat transfer medium flow through alternate channels. Because of the narrow channel between the plates, the fluid flows at high velocities and in a thin layer resulting in very high heat transfer rates per unit heat transfer surface area. The plate heat exchanger is mostly used for heating fluids to temperatures below the boiling point of water at atmospheric pressure. However there are units designed for high temperature service are commercially available. Plate heat exchangers are now used in virtually any application where tubular heat exchangers were previously commonly used. Newer

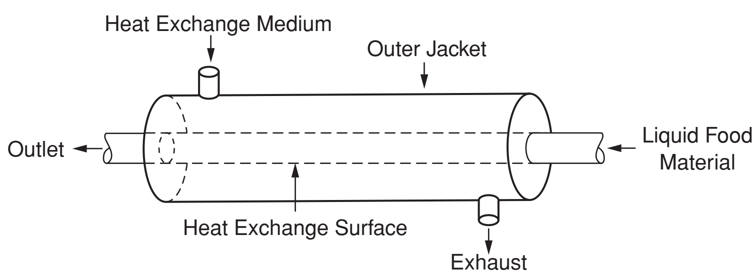

Figure 7.12 Diagram of a double-pipe heat exchanger.

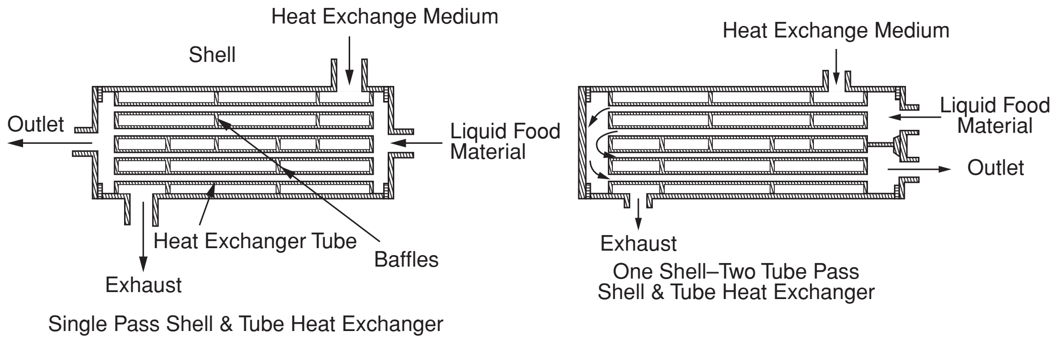

Figure 7.13 Diagram of a single-pass and a multiple-pass shell-and-tube heat exchanger.

designs have strength to withstand moderate pressure or vacuum. A major limitation is the inability to handle viscous liquids. Figure 7.14 shows a plate heat exchanger.

7.4.1 Heat Transfer in Heat Exchangers

The overall heat transfer coefficient in tubular exchangers is calculated using Equations (7.61) or (7.62). Because in heat exchangers the ΔT can change from one end of the tube to the other, a mean ΔT must be determined for use in Equation (7.57) to calculate q.

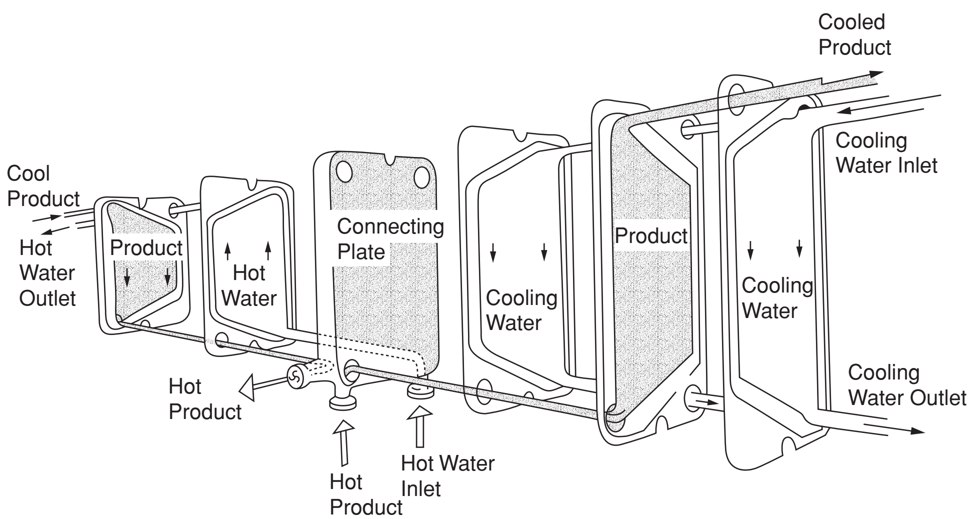

Figure 7.14 Diagram of a plat heat exchanger showing alternating paths of processed fluid and heat exchange medium.

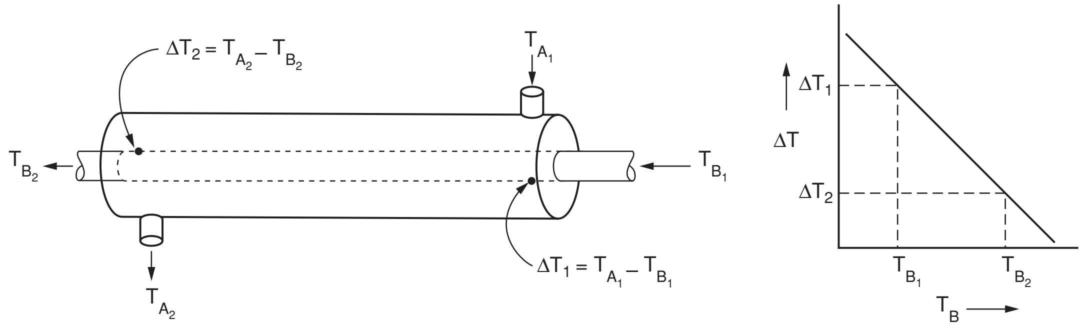

Figure 7.15 Diagram of the temperature profiles of fluid along the length of a heat exchanger tube.

7.4.2 The Logarithmic Mean Temperature Difference

Refer to Fig. 7.15 for the diagram and the representation of the symbols used in the following derivation.

If the Δ T changes linearly along the length of the heat exchanger relative to the temperature of one of the fluids, the slope of the line representing Δ T vs T_B is

$$\text{Slope} = \frac{d(\Delta\,T)}{dT_B} + \frac{\Delta\,T_2 - \Delta\,T_1}{T_{B2} - T_{B1}}$$

Rearranging:

$$dT_B = d\Delta T \frac{T_{B2} - T_{B1}}{\Delta T_2 - \Delta T_1}$$

The amount of heat transferred across any point in the exchanger is

$$q = U\,dA\Delta\,T$$

This heat transferred will cause a rise in product temperature, dT_B, and can be also expressed as:

$$q = m\,C_p\,dT_B$$

Equating:

$$U\,dA\,\Delta\,T = m\,C_p\,dT_B$$

Substituting the expression for dT_B:

$$U\,dA\,\Delta T = mC_p\left[\frac{d\,\Delta T(T_{B2} - T_{B1})}{(\Delta\,T_2 - \Delta\,T_1)}\right]$$

Rearranging:

$$mC_p(T_{B2} - T_{B1})\frac{d\Delta T}{\Delta T} = U(\Delta T_2 - \Delta T_1)(dA)$$

Integrating:

$$mC_p(T_{B2} - T_{B1}) \int_{\Delta T_1}^{\Delta T_2} \frac{d\Delta T}{\Delta T} = U(\Delta T_2 - \Delta T_1) \int_0^A dA$$

The limits are such that at the entrance of fluid B into the heat exchanger, $\Delta T = \Delta T_1$ and $A = 0$ at this point. When fluid B leaves the heat exchanger, $\Delta T = \Delta T_2$ and $A = A$ the total heat transfer area for the heat exchanger.

$$mC_p(T_{B2} - T_{B1}) \ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = UA(\Delta T_2 - \Delta T_1)$$

$mCp(T_{B2} - T_{B1}) = q$, the total amount of heat absorbed by fluid B. Thus:

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = UA\ \Delta T_L$$

Where: ΔT_L is the logarithmic mean ΔT expressed as:

$$\Delta T_L = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \tag{7.63}$$

When heat is exchanged between two liquids or between liquid and gas, the temperature of both fluids will be changing as they travel through the heat exchanger. If both fluids enter on the same end of the unit, flow will be in the same direction and this type of flow is cocurrent. Countercurrent flow exists when the fluids flow in opposite direction through the unit. True countercurrent or cocurrent flow would occur in double pipe, plate, and multiple tube single pass shell and tube heat exchangers. When true cocurrent or countercurrent flow exists, the rate of heat transfer can be calculated by:

$$q = U_iA_i\Delta T_L \qquad \text{or} \qquad q = U_oA_o\Delta T_L \tag{7.64}$$

The log mean temperature difference is evaluated between the inlet and exit ΔT values.

In multipass heat exchangers, the fluid inside the tubes will be traveling back ad forth across zones of alternating high and low temperatures. For this type of heat exchangers, a correction factor is used on the ΔT_L. The correction factor depends upon the manner in which shell fluid and tube fluid pass through the heat exchanger.

$$q = U_iA_iF\Delta T_L \quad \text{or} \quad q = U_oA_oF\Delta T_L \tag{7.65}$$

where F is the correction factor. The reader is referred to a heat transfer textbook or handbook for graphs showing correction factors to ΔT for multipass shell an tube heat exchangers.

Example 7.13. Applesauce is being cooled from 80°C to 20°C in a swept surface heat exchanger. The overall coefficient of heat transfer based on the inside surface area is 568 W/m$^2\cdot$ K. The applesauce has a specific heat of 3187 J/kg · K and is being cooled at the rate of 50 kg/h. Cooling water enters in countercurrent flow at 10°C and leaves the heat exchanger at 17°C. Calculate: (a) the quantity of cooling water required; (b) the required heat transfer surface area for the heat exchanger.

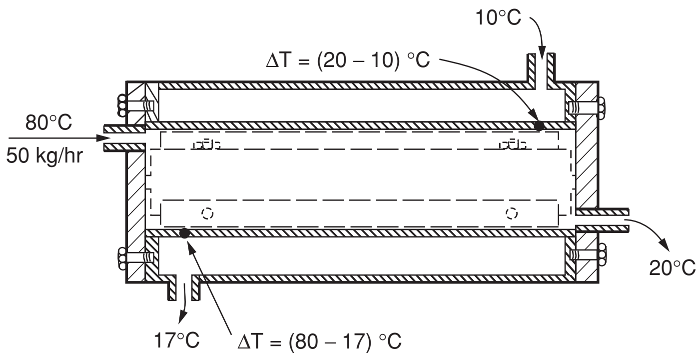

Figure 7.16 Diagram of a heat exchanger in a counterflow configuration for the fluids exchanging heat.

Solution:

The diagram of the heat exchanger in countercurrent flow is shown in Fig. 7.16.

(a) The rate of heat transfer =

$$50\frac{\text{kg}}{\text{h}} \cdot 3187\frac{\text{J}}{\text{kg}\cdot\text{K}} \cdot (80 - 20)^{\circ}\text{K} = 9,561,000\frac{\text{J}}{\text{h}} = 2.656\,\text{kW}$$

Let x = quantity of water used. The specific heat of water is 4186 J/kg · K

$$x(4186)(17 - 10) = 9561000$$
$$x = \frac{9,561,000}{4186(7)} = 326\,\text{kg/h}$$

(b) $\Delta T_2 = 20 - 10 = 10^{\circ}\text{C}$

$\Delta T_1 = 80 - 17 = 63^{\circ}\text{C}$

Using Equation (7.63):

$$\Delta T_L = \frac{63 - 10}{\ln 63/10} = \frac{53}{\ln 6.3} = 28.8$$

The rate of heat transfer is calculated using Equation (7.64).

$$q = U_i A_i\ \Delta T_L = 568\,A_i\ (28.8)$$
$$(568)(Ai)(28.8) = 2656$$
$$A_i = \frac{2656}{568(28.8)} = 0.162\ \text{m}^2$$

7.5 LOCAL HEAT TRANSFER COEFFICIENTS

A major problem in heat transfer is the estimation of heat transfer coefficients to be used for design purposes. These values of h must be determined from the properties of the fluid and the geometry of the system.

7.5.1 Dimensionless Quantities

The use of dimensionless quantities arises from the principle of similarity. Equations that describe different systems having similar characteristics can be superimposed on each other to form a single expression suitable for all systems. Thus, if the physical characteristics of a fluid and the conditions that exist in an experiment are expressed in terms of dimensionless quantities, it will be possible to extrapolate results of an experiment to other fluids and other conditions. The principle of similarity makes it unnecessary to experimentally establish equations for heat transfer to each fluid. A general correlation equation will be suitable for all fluids. The determination of the different dimensionless quantities involved in relationships between variables describing a system is done by a method called dimensional analysis. In dimensional analysis, an equation relating the various variables is first assumed, and by performing an analysis of the base dimensions of the variables to make the equation dimensionally consistent, specific groupings of the variables are formed. The following dimensionless quantities have been identified and used in correlations involving the heat transfer coefficient.

Nusselt number (Nu): This expression involves the heat transfer coefficient (h), the characteristic dimension of the system (d), and the thermal conductivity of the fluid (k). This dimensionless expression may be considered as the ratio of the characteristic dimension of a system and the thickness of the boundary layer of fluid that would transmit heat by conduction at the same rate as that calculated using the heat transfer coefficient.

$$\mathrm{Nu} = h\frac{d}{k} \tag{7.66}$$

Reynolds number (Re): This expression involves the characteristic dimension of the system (d), the velocity of the fluid (V), the density (ρ), and the viscosity (μ). It may be considered as the ratio of inertial forces to the frictional force. The Reynolds number has been discussed in Chapter 6.

Prandtl number (Pr): This expression involves the specific heat (C_p), the viscosity (μ), and the thermal conductivity (k). It may be considered as the ratio of rate of momentum exchange between molecules and the rate of energy exchange between molecules that lead to the transfer of heat.

$$\mathrm{Pr} = \frac{C_P\ \mu}{k} \tag{7.67}$$

Grashof number (Gr): This quantity involves the characteristic dimension of a system (d), the acceleration due to gravity (g), the thermal expansion coefficient (β), the density of the fluid (ρ), the viscosity (μ), and the temperature difference ΔT between a surface and the fluid temperature beyond the boundary layer. This number may be considered as a ratio of the force of gravity to buoyant forces that arise as a result of a change in temperature of a fluid.

$$\mathrm{Gr} = \frac{d^3\ g\ \beta\ \rho^2\ \Delta T}{\mu^2} \tag{7.68}$$

Peclet number (Pe): This dimensionless number is the product of the Reynolds number and the Prandtl number.

$$Pe = Re\ Pr = \frac{\rho\, V\, C_p\, d}{k} \tag{7.69}$$

Rayleigh number (Ra): This dimensionless number is the product of the Grashof number and the Prandtl number.

$$Ra = Gr\ Pr = \frac{d^3\ g\, \beta\ C_p \rho^2\ \Delta T}{\mu\, k} \tag{7.70}$$

Graetz number (Gz): This is similar to the Peclet number. It was derived from an analytical solution to the equations of heat transfer from a surface to a fluid flowing along that surface in laminar flow. The Graetz number is

$$Gz = \frac{\pi}{4}\left[Re \cdot Pr \cdot \left(\frac{d}{L}\right)\right] = \frac{\dot{m}\ C_p}{kL} \tag{7.71}$$

where $\overline{m}$ is the mass rate of flow, kg/s.

7.5.2 Equations for Calculating Heat Transfer Coefficients

These equations generally take the form:

$$Nu = f\left(Re, Pr, \left(\frac{L}{d}\right), Gr, \left(\frac{\mu}{\mu_w}\right)\right) \tag{7.72}$$

The Grashof number is associated with free convection, and the length to diameter ratio (L/d) appears when flow is laminar. When calculating these dimensionless quantities, the thermophysical properties of fluids at the arithmetic mean temperature at the inlet and exit are used. The viscosity of fluid at the wall (μ_w) affects heat transfer and is included in the correlation equation to account for the difference in cooling and heating processes. Most of the English literature on correlation equations for heat transfer are based on the general expression:

$$Nu = \alpha(Re)^{\beta}(Pr)^{\gamma}\left[\frac{L}{d}\right]^{\delta} \tag{7.73}$$

where α, β, γ, and δ are constants obtained from correlation analysis of experimental data.

A number of correlation equations suitable for use under various conditions have been published. A summary of equations suitable for use under commonly encountered conditions in food processing is given in Appendix Table A.12. In order to illustrate the use of these equations in the design of a heating or cooling system, some equations are also given in this chapter.

7.5.2.1 Simplified Equations for Natural Convection to Air or Water

For natural convection to air or water, Table 7.3 lists simplified equations based on those given by McAdams (1954).

Table 7.3 Equations for Calculating Heat Transfer Coefficients in Free Convection from Water or Air

		Value of C for:	
Surface Conditions	*Equation*	*Air*	*Water*
Horizontal cylinder heated or cooled	$h = C\left(\frac{\Delta T}{D}\right)^{0.25}$	1.396	291.1
Fluid below heated horizontal plate	$h = C(\Delta T)^{0.25}$	2.4493	
Fluid below heated horizontal plate	$h = C(\Delta T)^{0.25}$	1.3154	
Fluid above cooled horizontal plate	$h = C(\Delta T)^{0.25}$	1.3154	
Fluid below cooled horizontal plate	$h = C(\Delta T)^{0.25}$	2.4493	
Vertical cylinder heated or cooled	$h = C\left(\frac{\Delta T}{D}\right)^{0.25}$	1.3683	127.1
Vertical plate heated or cooled	$h = c\left(\frac{\Delta T}{L}\right)^{0.25}$	1.3683	127.1

Source: Calculated from values reported in McAdams, W. H. 1954. *Heat Transmission Co.,* New York. 3rd ed. McGraw-Hill Book Co., New York.

7.5.2.2 Fluids in Laminar and Turbulent Flow Inside Tubes

Equation (7.74), used for laminar flow, was originally derived by Leveque and is discussed in a number of heat transfer textbooks. Equation (7.75) is another form of Equation (7.74). Equation (7.76), commonly referred to as the Dittus-Boelter equation, is used for turbulent flow.

$$\mathrm{Nu} = 1.615\left[\mathrm{Re}\,\mathrm{Pr}\left(\frac{\mathrm{d}}{\mathrm{L}}\right)\right]^{0.33}\left(\frac{\mu}{\mu_{\mathrm{w}}}\right)^{0.14} \tag{7.74}$$

Sieder and Tate (1936) introduced the use of the ratio of viscosity of fluid bulk and that at the wall as a multiplying factor to account for the difference in the heat transfer coefficient between a fluid and a heated or a cooled surface.

$$\mathrm{Nu} = 1.75(\mathrm{Gz})^{0.33}\left(\frac{\mu}{\mu_{\mathrm{w}}}\right)^{0.14} \tag{7.75}$$

$$\mathrm{Nu} = 0.023(\mathrm{Re})^{0.8}(\mathrm{Pr})^{0.33}\left(\frac{\mu}{\mu_{\mathrm{w}}}\right)^{0.14} \tag{7.76}$$

7.5.2.3 Heat Transfer to Non-Newtonian Fluids in Laminar Flow

Metzner et al. (1957) used Leveque's solution to heat transfer to a moving fluid in laminar flow as a basis for non-Newtonian flow heat transfer. They defined the delta function, a factor $\Delta^{0.33}$, as the ratio $\mathrm{Nu_n}/\mathrm{Nu}$ (non-Newtonian Nusselt number/Newtonian Nusselt number).

Using the delta function as a multiplying factor for the Newtonian Nusselt number, Equation (7.74) can be used for non-Newtonian fluids as follows:

$$\mathrm{Nu} = 1.75\Delta^{0.33}\mathrm{Gz}^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{7.77}$$

When the flow behavior index is greater than 0.4 for all values of Gz, $\Delta^{0.33} = [(3n + 1)/4n]^{0.33} = \delta^{0.33}$. When $n < 0.4$, $\Delta^{0.33}$ is determined as follows:

$$\Delta^{0.33} = -0.24\,n + 1.18 \quad \mathrm{Gz} = 5,\ 0 < n < 0.4 \tag{7.78}$$

$$\Delta^{0.33} = -0.60\,n + 1.30 \quad \mathrm{Gz} = 10,\ 0 < n < 0.4 \tag{7.79}$$

$$\Delta^{0.33} = -0.72\,n + 1.40 \quad \mathrm{Gz} = 15,\ 0 < n < 0.4 \tag{7.80}$$

$$\Delta^{0.33} = -0.35\,n + 1.57 \quad \mathrm{Gz} = 25,\ 0 < n < 0.4. \tag{7.81}$$

Equation (7.81) can also be used when $\mathrm{Gz} > 25$, $0.1 < n < 0.4$

For $\mathrm{Gz} > 25$ and $n < 0.1$, the relationship between $\Delta^{0.33}$ and n is nonlinear and the reader is referred to the paper of Metzner et al. (1957) for determination of the delta function. Most food fluids will have $n > 0.1$, therefore the equations for the delta function given above should be adequate to cover most problems encountered in food processing.

7.5.2.4 Adapting Equations for Heat Transfer Coefficients to Non-Newtonian Fluids

The approach of Metzner et al. (1957) to adapting correlation equations for heat transfer coefficients derived for Newtonian fluids to non-Newtonian fluids will allow the use of practically any of the extensive correlations derived for Newtonian fluids for systems involving non-Newtonian fluids. For non-Newtonian fluids, the viscosity is not constant in the radial direction of the pipe. It will be possible, however, to use the equations derived for Newtonian fluids by using an equivalent viscosity for the non-Newtonian fluid that will cause the same pressure drop at the flow rate under consideration. A similar approach was used by Metzner et al. (1957) for heat transfer to non-Newtonian fluids in turbulent flow.

For a non-Newtonian fluid that follows the Power Law equation:

$$\tau = K\,(\gamma)^n$$

where τ = the shear stress, γ = the shear rate, K = the consistency index and n = the flow behavior index.

The Reynolds number at an average rate of flow V is

$$\mathrm{Re} = \frac{8(V)^{2-n}R^n\rho}{K[(3n+1)/n]^n}$$

The viscosity to be used in the Prandtl number and the Sieder-Tate viscosity correction term for the fluid bulk is determined as follows.

For a Newtonian fluid:

$$\mathrm{Re} = \frac{DV\rho}{\mu}$$

Substituting the Reynolds number of the non-Newtonian fluid:

$$\mu = \frac{DV\rho}{Re}$$

$$= K\left(\frac{3n+1}{n}\right)^n \left[\frac{2RV\rho}{8V^{2-n}R^n\rho}\right]$$

$$\mu = \frac{DV\rho}{8(V)^{2-n}R^n\rho/[K(3+1/n)^n]}$$

$$\mu = \frac{K}{4}\left[\frac{3n+1}{n}\right]^n (R)^{1-n}(V)^{n-1} \tag{7.82}$$

At the wall, the viscosity μ_w used in the Sieder-Tate viscosity correction term is determined from the apparent viscosity (Eq. 6.6, Chapter 6) as follows:

$$\mu_w = K(\gamma_w)^{n-1}$$

The shear rate at the wall, γ_w, is calculated using the Rabinowitsch-Mooney equation (Eq. 6.18, Chapter 6).

$$\gamma_w = \frac{8V}{D}\left[\frac{3n+1}{4n}\right] = \frac{2V}{D}\left[\frac{3n+1}{n}\right]$$

$$\mu_w = K\left(\frac{2V}{D}\right)^{n-1}\left[\frac{3n+1}{n}\right]^{n-1} \tag{7.83}$$

Equations (7.82) and (7.83) are the viscosity terms for non-Newtonian fluids used in the correlation equations for heat transfer coefficients to Newtonian fluids.

As an example, Equation (7.74) will be used to determine an equivalent equation for non-Newtonian fluids. Because Leveque's derivation of Equation (7.74) was based on the fluid velocity profile at the boundary layer near the pipe wall, the viscosity term in the Reynolds number should be the apparent viscosity at the shear rate existing at the pipe wall. The viscosity term in the Prandtl number is based on viscous dissipation and conduction in the bulk fluid and should be the apparent viscosity based on the average velocity. Substituting Equation (7.83) for the μ term in the equation for the Reynolds number Φ and Equation (7.82) for the Φ term in the equation for the Prantl number, The Nusselt number expression of Equation (7.74) becomes:

$$Nu = 1.615\left[\left[\frac{DV\rho}{K[2V/D]^{n-1}[(3n+1)/n]^{n-1}}\right]\right.$$
$$\left.\times\left[\frac{C_p(K/4)[(3n+1)/n]^n[D/2]^{1-n}V^{n-1}}{k}\left[\frac{D}{L}\right]\right]\right]^{0.33}\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

Simplifying:

$$Nu = 1.615\left[\frac{3n+1}{4n}\right]^{0.33}\left[\frac{D^2V\rho C_p}{kL}\right]^{0.33}\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

Substituting the mass rate of flow calculated from the volumetric rate of flow and the density:

$$\dot{m} = \frac{\pi D^2}{4} \cdot V \cdot \rho \qquad D^2 V\rho = \frac{4}{\pi}\dot{m}$$

$$Nu = 1.615 \left[\frac{4}{\pi}\right]^{0.33} \left[\frac{\dot{m}C_p}{kL}\right]^{0.33} \left[\frac{3n+1}{4n}\right]^{0.33} \left[\frac{\mu}{\mu_w}\right]^{0.14}$$

This expression is similar to Equation (7.77) except for the value of the constant.

Example 7.14. Calculate the rate of heat loss from a 1.524 m inside diameter horizontal retort, 9.144 m long. Steam at 121°C is inside the retort. Ambient air is at 25°C. The retort is made out of steel ($k = 42\ W/m \cdot K$) and has a wall thickness of 0.635 cm.

Solution:

The heat transfer coefficient from the outside surface to ambient air will be controlling the rate of heat transfer; therefore, variations in the steam side heat transfer coefficient will have little effect on the answer. Assume the steam side heat transfer coefficient is 6000 $W/(m^2 \cdot K)$.

The system is a horizontal cylinder with a vertical plate at each end. The outside heat transfer coefficient can be calculated using the appropriate equations from Table 7.3.

For a horizontal cylinder to air:

$$h = 1.3196 \left[\frac{\Delta T}{D_o}\right]^{0.25}$$

For a vertical plate to air:

$$h = 1.3683 \left[\frac{\Delta T}{L}\right]^{0.25}$$

ΔT is evaluated from the outside wall temperature. The problem can be solved using a trial and error procedure by first assuming a heat transfer coefficient, calculating a wall temperature and recalculating the heat transfer coefficient until the assumed values and calculated values converge.

The magnitude of the heat transfer coefficient in free convection to air is of the order 5 $W/m^2 \cdot K$. Equation (7.62) can be used to calculate an overall heat transfer coefficient for the cylindrical surface.

$$\frac{1}{U_i} = \frac{r_i}{r_o h_0} + \frac{r_i \ln r_o/r_i}{k} + \frac{1}{h_i} = 0.1984 + 0.00015 + 0.000167$$

$$= \frac{0.762}{0.76835(5)} + \frac{0.762 \ln(0.76835/0.762)}{42} + \frac{1}{6000}$$

$$U_i = 5.03\ W/(m^2 AK)$$

$$U_i A_i \Delta T = h_o A o \Delta t_w$$

$$U_i(2\pi r_i L)\Delta T = h_o(2\pi r_o L)\Delta t_w$$

$$U_i r_i \Delta T = h_o r_o \quad \Delta t_w$$

$$\Delta T_w = \frac{U_i r_i \Delta T}{h_o r_o} = \frac{5.03(0.762)(121-25)}{5(0.76835)} = 95.78°C$$

$$h_o = 1.3196\left(\frac{\Delta T_w}{D_o}\right)^{0.25} = 1.3196\left(\frac{95.78}{1.5367}\right)^{0.25} = 3.71 W/(m^2 K)$$

The calculated value is less than the assumed h_o. Use this calculated value to recalculate U_i. All the terms in the expression for $1/U_i$, previously used are the same except for the first term. Assume $h_o = 3.71 W/(m^2 \cdot K)$.

$$\frac{1}{U_i} = \frac{0.762}{0.76835(3.71)} + 0.00015 + 0.000167$$

$$U_i = 3.7365\ W/(m^2\,K)$$

$$\Delta T_w = \frac{3.7365(0.762)(121 - 25)}{3.71(0.76835)} = 95.89°C$$

$$h_o = 1.3196(\frac{95.89}{1.5367})^{0.25} = 3.71 W/(m^2\ K)$$

Because the assumed and calculated values are the same, the correct h_o must be $h_o = 3.71 W/(m^2 \cdot K)$ and:

$$q = U_i A_i \quad \Delta T = 3.7365(\pi)(1.524)(9.144)(121 - 25) = 15,696\ W$$

For the ends, examination of the equation for the heat transfer coefficient reveal that h with vertical plates is approximately 4% higher than h with horizontal cylinders for the same ΔT if L is approximately the same as D. Assume h = 3.71 (1.04) = 3.86 W/(m$^2 \cdot$ K). Because the end is a vertical flat plate, U can be calculated using Equation (7.59).

$$\frac{1}{U} = \frac{1}{3.86} + \frac{0.00635}{42} + \frac{1}{6000}$$

$$U = 3.855\ W/(m^2 \cdot K)$$

$$UA\Delta T = h_o\ A\ \Delta\ T_w$$

$$\Delta T_w = \frac{U\Delta T}{h_o} = \frac{3.855(121 - 25)}{3.86} = 95.88$$

$$h_o = 1.3683 \left[\frac{95.87}{1.5367}\right]^{0.25} = 3.846\ W/(m^2 \cdot K)$$

The assumed and calculated values are almost the same. Use $h_o = 3.846\ W/(m^2 \cdot K)$:

$$\frac{1}{U} = \frac{1}{3.846} + 0.000156 + 0.000167$$

$$U = 3.835\ W/(m^2 \cdot K)$$

Because there are two sides, the area will be: $A = 2\pi R^2$

$$q = U\ A\ \Delta\ T = (3.835)(\pi)(1.5367)^2(2)(121 - 25) = 5463W$$

Total heat loss = 15,704 + 5463 = 21,167 W.

Example 7.15. Calculate the overall heat transfer coefficient for applesauce heated from 20°C to 80°C in a stainless steel tube 5 m long with an inside diameter of 1.034 cm and a wall thickness of 2.77 mm. Steam at 120°C is outside the tube. Assume a steam side heat transfer coefficient of 6000 W/(m$^2 \cdot$ K). The rate of flow is 0.1 m/s. Applesauce has a density of 995 kg/m^3 and this density is assumed to be constant with temperature. The value for n is 0.34 and the values for K are 11.6 at 30°C and 9.0 at 82° C in Pa · s units. Assume K changes with temperature according to an Arrhenius type relationship as follows:

$$\log K = A + \frac{B}{T}$$

where A and B are constants and T is the absolute temperature. The thermal conductivity may be assumed to be constant with temperature at 0.606 W/(m · K). The specific heat is 3817 J/(kg · K). The thermal conductivity of the tube wall is 17.3 W/(m · K).

First, determine the temperature dependence of K. B is the slope of a plot of log K against 1/T.

$$B = \frac{\log 11.6 - \log 9.0}{(1/303 - 1/355)} = \frac{1.064458 - 0.954243}{0.0033 - 0.00282} = 227.99$$

$$A = \log 11.6 - \frac{227.99}{303} = 1.064458 - 0.752433 = 0.3120$$

$$\log K = 0.3143 + \frac{227.99}{T}$$

The arithmetic mean temperature for the fluid is $(20 + 80)/2 = 50°C$. At this temperature, $K = \log^{-1}(0.3143 + 227.3/323) = 10.42$.

$$G = \frac{0.1\,m}{s} \frac{1}{\pi(0.00517)^2\ m^2} = 1190.88\,kg/(s \cdot m^2)$$

The equivalent Newtonian viscosity is calculated using Equation (7.82).

$$\mu = \frac{10.42}{4}\left(\frac{3(0.34)+1}{0.34}\right)^{0.34}(0.00517)^{1-0.34}(0.1)^{0.34-1}$$

$$= \frac{10.42}{4}(1.8328)(0.030967)(4.5709) = 0.676\ Pa \cdot s$$

$$Re = \frac{0.01034(0.1)(995)}{0.676} = 1.521; \quad \text{flow is laminar}$$

Equation (7.77) must be used, because $n < 0.4$ and the delta function deviates from $\delta = (3n + 1)/4n$.

Solving for the Graetz number:

$$\dot{m} = \pi\left(\frac{D^2}{4}\right)(V)(\rho) = \pi(0.00517)^2(0.1)(995) = 0.008355$$

Using Equation (7.79) for $n < 0.4$ and $Gz = 10$:

$$Gz = \frac{0.008355(3817)}{[(0.606)(5)]} = 10.52$$

$$\Delta^{0.33} = -0.6(0.34) + 1.30 = 1.096$$

The Nusselt number is

$$Nu = 1.75(1.096)(10.525)^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$= 4.170\left(\frac{\mu}{\mu_w}\right)^{0.14} = \frac{hD}{k}$$

Solving for h:

$$h = \frac{4.17[\mu/\mu_w]^{0.14}(0.606)}{0.01034} = 244.39\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

Assume most of the temperature drop occurs across the fluid side resistance. Assume that $T_w = 116°C$.

$$K \text{ at } 116°C = \log^{-1}(0.3126 + 227.99/389) = 7.91\ Pa \cdot s$$

Using Equation (7.83):

$$\mu_w = 7.91\left[\frac{(2)(0.1)}{0.01034}\right]^{0.34-1}\left[\frac{3(0.34)+1}{0.34}\right]^{0.34-1} = 7.91(0.141548)(0.30849) = 0.3454$$

$$h_i = 244.39\left[\frac{0.676}{0.3454}\right]^{0.14} = 268.48W/(m^2 \cdot \leq K)$$

Using Equation (7.62) for the overall heat transfer coefficient:

$$\frac{1}{U_i} = \frac{r_i}{r_o h_o} + \frac{r_i \ln r_o/r_i}{k} = \frac{1}{h_i}$$

$$r_i = 0.00517; \quad r_o = 0.00517 + 0.00277 = 0.00794$$

$$\frac{1}{U_i} = \frac{0.00517}{0.00794(6000)} + \frac{0.00517\ln(0.00794/0.00517)}{17.3} + \frac{1}{268.48}$$

$$= 0.000109 + 0.000128 + 0.003725 = 0.003962; \quad U_i = 252.4$$

Check if wall temperature is close to assumed temperature.

$$U_i A_i\ \Delta T = hA_i\ \Delta T_w$$

$$\Delta T_w = \frac{U_i A_i \Delta T}{h_i A_i} = \frac{252.42}{268.48}(120 - 50) = 65.8$$

$$T_w = 50 + 65.8 = 115.8$$

This is close to the assumed value of 116°C, therefore:

$$h_i = 268.48\ W/(m^2.K) \text{ and } U_i = 252.42\ W/(m^2 \cdot K)$$

Example 7.16. The applesauce in Example 7.14 is being cooled at the same rate from 80°C to 120°C in a double pipe heat exchanger with water flowing in the annular space outside the tube at a velocity of 0.5 m/s. The inside diameter of the outer jacket is 0.04588 m. Water enters at 8°C and leaves at 16°C. Calculate the food side and the water side heat transfer coefficient and the overall heat transfer coefficient.

Solution:

First calculate the water side heat transfer coefficient. At a mean temperature of 12°C water has the following properties: $k = 0.58\,W/(m \cdot K)$, $\mu = 1.256 \times 10^{-3}\ Pa \cdot s$; $\rho = 1000 kg/m^3$; $C_p = 4186\,J/(kg \cdot K)$; and $D_o = 0.01588\,m$. For an annulus, the characteristic length for the Reynolds number is 4(cross-sectional area/wetted perimeter).

$$D = \frac{4(\pi)\left(d_2^2 - d_1^2\right)/4}{(\pi)(d_1 + d_2)} = d_2 - d_1 = 0.03$$

$$Re = \frac{D_o\overline{V}\rho}{\mu} = \frac{(0.030)(0.5)(1000)}{1.256 \times 10^{-3}} = 11,942$$

$$Pr = \frac{C_p\mu}{k} = \frac{4186(1.256 \times 10^{-3})}{0.58} = 9.06$$

For water in turbulent flow in an annulus, Monod and Allen's equation from Appendix Table A.12 with the viscosity factor is

$$\frac{hD_o}{k} = 0.02\,Re^{0.8}\,Pr^{0.33}\left[\frac{d_2}{d_1}\right]^{0.53}\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

$$= 0.02(11942)^{0.8}(9.06)^{0.33}\left[\frac{0.04588}{0.01588}\right]^{0.53}\left[\frac{\mu}{\mu_w}\right]^{0.14} = 2565\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

Assume that $T_w = 2°C$ higher than the arithmetic mean fluid temperature. $T_w = 15°C$; $\mu_w = 1.215 \times 10^{-3}$ Pa · s.

Now, calculate the food fluid side heat transfer coefficient. Since the fluid arithmetic mean

$$h_o = \frac{0.58}{0.03}(0.02)(1827)(2.069)(1.754)\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

$$h_o = 2565\left[\frac{1.256}{1.215}\right]^{0.14} = 2577\,W/(m^2 \cdot K)$$

temperature is the same as in Example 7.14.

$$h_i = 256.7\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

The overall ΔT between the water and the applesauce based on the mean temperature is

$50 - 12 = 38°C$. $T_w = 15°C$.

$$K_w = \log^{-1}\left(\frac{0.3126 + 227.99}{288}\right) = 12.69\ Pa \cdot s$$

From Example 7.14: $\mu = 0.676$ Pa · s. Using Equation (7.83):

$$\mu_w = 12.69\left[\frac{2(0.1)}{0.01034}\right]^{0.34-1}\left[\frac{3(0.34)+1}{0.34}\right]^{0.34-1} = 0.554$$

$$h_i = 256.7\left[\frac{0.676}{0.554}\right]^{0.14} = 263.95\ W/(m^2 \cdot K)$$

Calculating for U_i using Equation (7.62):

$$\frac{1}{U_i} = \frac{0.00517}{0.00794(2577)} + \frac{0.00517\ln(0.00794/0.00517)}{17.3} + \frac{1}{263.95}$$

$$= 0.000253 + 0.000128 + 0.003789 = 0.004170$$

$$U_i = 239.83\,W/(m^2 \cdot K)$$

Solving for the wall temperature:

$$\Delta T_w = \frac{U_i \Delta T}{h_i} = \frac{(239.83)(38)}{263.95} = 34.5°C$$

$$T_w = 50 - 34.5 = 15.5°C$$

The small difference in the calculated and assumed wall temperatures will not alter the calculated h_i and h_o values significantly; therefore, a second iteration is not necessary. The values for the heat transfer coefficients are

Water side: $h_o = 2577\ W/(m^2 \cdot K)$
Food fluid side: $h_i = 263.95\ W/(m^2 \cdot K)$
Overall heat transfer coefficient: $U_i = 239.83\ W/(m^2 \cdot K)$

7.6 UNSTEADY-STATE HEAT TRANSFER

Unsteady-state heat transfer occurs when food is heated or cooled under conditions where the temperature at any point within the food or the temperature of the heat transfer medium changes with time. In this section, procedures for calculating temperature distribution within a solid during a heating or cooling process will be discussed.

7.6.1 Heating of Solids Having Infinite Thermal Conductivity

Solids with very high thermal conductivity will have a uniform temperature within the solid. A heat balance between the rate of heat transfer and the increase in sensible heat content of the solid gives:

$$mC_p \frac{dT}{dt} = hA(T_m - T)$$

Solving for T and using the initial condition, $T = T_o$ at $t = 0$:

$$\ln\left(\frac{T_m - T}{T_m - T_o}\right) = \frac{hA}{mC_p} t \tag{7.84}$$

Equation (7.84) shows that a semi-logarithmic plot of $\theta = (T_m - T)/(T_m - T_o)$ against time will be linear with a slope $-hA/mC_p$. If a solid is heated or cooled, the dimensionless temperature ratio, θ, plotted semi-logarithmically against time, will show an initial lag before becoming linear. If the extent of the lag time is defined such that the point of intersection of the linear portion of the heating curve with the ordinate is defined, it will be possible to calculate point temperature changes in a solid for long heating times, from an empirical value of the slope of the heating curve. This principle is used in thermal process calculations for foods in Chapter 8.

Fluids within a well-stirred vessel exchanging heat with another fluid that contacts the vessel walls will change in temperature as in Equation (7.84). A well-mixed steam-jacketed kettle used for cooking foods is an example of a system where temperature change will follow Equation (7.84). For a well-mixed steam-jacketed kettle, U, the overall heat transfer coefficient between the steam and the fluid inside the kettle, will be used instead of the local heat transfer coefficient h used in deriving equation 84.

Example 7.17. A steam-jacketed kettle consists of a hemispherical bottom having a diameter of 69 cm and cylindrical side 30 cm high. The steam jacket of the kettle is over the hemispherical bottom only. The kettle is filled with a food product that has a density of 1008 kg/m^3 to a point 10 cm from the rim of the kettle. If the overall heat transfer coefficient between steam and the food in the jacketed part of the kettle is $1000W/(m^2 \cdot K)$, and steam at 120°C is used for heating in the jacket, calculate the time for the food product to heat from 20°C to 98°C. The specific heat of the food is 3100 J/(kg · K).

Solution:

The surface area of a hemisphere is $(1/2)\pi d^2$; the volume is $0.5(1/6)\pi d^3$. Use Equation (7.84) to solve for t. A = area of hemisphere with d = 0.69 m. $A = 0.5(\pi)(d^2) = 0.7479\ m^2$.

$$
\begin{aligned}
\text{Volume} &= 0.5(0.1666)(n)(.693) + n[(0.69)(0.5)]2(0.30 - 0.10) \\
&= 0.08597 + 0.07478 = 0.16075\ m^3 \\
&= 0.16075m^3(1008\ kg/m^3) = 162.04\ kg \\
T_m &= 120;\ T_o = 20;\ U = 1000;\ C_p = 3100
\end{aligned}
$$

$$
\ln\left(\frac{120-98}{120-20}\right) = -\frac{1000(0.7479)}{162.04(3100)}\,t = -0.001489\,t
$$

$$
t = \frac{-\ln(0.22)}{0.001489} = 1016.9\,s = 16.95\,min
$$

7.6.2 Solids with Finite Thermal Conductivity

When k is finite, a temperature distribution exists within the solid. Heat transfer follows Fourier's second law, which was derived in the section "Heat Transfer by Conduction" (Eq. 7.19) for a rectangular parallelopiped. For other geometries, the following differential equations represent the heat balance (Carslaw and Jaeger, 1959).

For cylinders with temperature symmetry in the circumferential direction:

$$
\frac{\delta T}{\delta t} = \alpha\left(\frac{\delta^2 T}{\delta r^2} + \frac{\delta T}{r\delta r} + \frac{\delta^2 T}{\delta Z^2}\right) \qquad (7.85)
$$

For spheres with symmetrical temperature distribution:

$$
\frac{\delta T}{\delta t} = \alpha\left[\frac{\delta^2(rT)}{r\delta r^2} + \frac{1}{r^2 \sin\phi} + \frac{\phi}{\delta\phi}\left(\sin\phi\frac{\delta T}{\delta\phi}\right)\right] \qquad (7.86)
$$

Thermal diffusivity: $\alpha = k/(\rho C_p)$. An excel program for calculating α from values of k, ρ, and C_p determined from food composition is given in Appendix A11.

These equations may be solved analytically by considering one-dimensional heat flow and obtaining a composite solution by a multiplicative superposition technique. Thus, the solution for a brick-shaped solid will be the product of the solutions for three infinite slabs, and that for a finite cylinder will be the product of the solution for an infinite cylinder and an infinite slab. Analytical solutions are not easily obtained for certain conditions in food processing, such as an interrupted process where a change in boundary conditions occurred at a midpoint in the process, or when the boundary temperature is an undetermined function of time. Equations (7.19), (7.85), and (7.86) may be solved using a finite difference technique if analytical solutions are not suitable for the conditions specified. Techniques for solving partial differential equations are discussed and analytical solutions are given in Carslaw and Yaeger (1959).

7.6.3 The Semi-Infinite Slab with Constant Surface Temperature

A semi-infinite slab is defined as one with infinite width, length, and depth. Thus, heat is transferred in only one direction, from the surface toward the interior. This system is also referred to as a "thick solid" and under certain conditions, such as at very short times of heating, the surface of a solid and a point very close to the surface would have the temperature response of a semi-infinite solid to a sudden change in the surface conditions. Equation (7.19) may be solved using the boundary conditions: at time 0, the slab is initially at T_o and the surface is suddenly raised to temperature T_s. The temperature T at any point x, measured from the surface, expressed as a dimensionless temperature ratio, θ, is

$$\theta = \mathrm{erf}\left[\frac{x}{(4\,\alpha\,t)^{0.5}}\right] \tag{7.87}$$

$\theta = (T_s - T)/(T_s - T_O)$ and erf is the error function. The error function is a well studied mathematical function, and values are found in standard mathematical tables. Values of the error function are given in Table 7.4.

The error function approaches 1.0 when the value of the argument is 3.6. In a solid, the point where erf = 1 is undisturbed by the heat applied at the surface. Thus, a penetration depth may be calculated, beyond which the thermal conditions are undisturbed. If this penetration depth is less than the half thickness of a finite slab, the temperature distribution near the surface may be approximated by Equation (7.87). A finite body may exhibit a thick body response if the penetration depth is much less than the half thickness. The thick body response to a change in surface temperature is the temperature distribution expressed by Equation (7.87) from the surface ($x = 0$) to a point in the interior s distant from the surface, when $\delta = 3.8(4\alpha t)^{0.5}$ is much less than the half-thickness of a finite solid.

If the surface heat transfer coefficient is finite, the solution to Equation (7.19) with the boundary condition: $T = T_o$ at time zero, heat is transferred at the surface from a fluid at temperature T_m, with

Table 7.4 Values of the Error Function.

x	*erf (x)*	*x*	*erf (x)*	*x*	*erf (x)*
0	0	0.70	0.677801	1.7	0.983790
0.05	0.056372	0.75	0.711156	1.8	0.989091
0.10	0.112463	0.80	0.742101	1.9	0.992790
0.15	0.157996	0.85	0.770668	2.0	0.995322
0.20	0.222703	0.90	0.796908	2.2	0.998137
0.25	0.276326	0.95	0.820891	2.4	0.999311
0.30	0.328627	1.00	0.842701	2.6	0.999764
0.35	0.37938	1.1	0.880205	2.8	0.999925
0.40	0.428392	1.2	0.910314	3.0	0.999978
0.45	0.47548	1.3	0.934003	3.2	0.999994
0.50	0.520500	1.4	0.952790	3.4	0.999998
0.55	0.563323	1.5	0.966105	3.6	1.00000
0.60	0.603856	1.6	0.976348	3.8	1.00000
0.65	0.64202			4.0	1.00000

Source: Kreyzig, E. 1963. Advanced engineering Mathematics, John Wiley & Sons, NY.

a heat transfer coefficient h:

$$\theta = \text{erf}\left[\frac{x}{\sqrt{(4\alpha t)}} + [e]^{hx/k+(h/k)^2\alpha t}\left[\text{erfc}\left[\frac{x}{\sqrt{(4\alpha t)}} + \text{h over k}\sqrt{(\alpha t)}\right]\right]\right] \tag{7.88}$$

where erfc[F(x)] = 1 − erf[F(x)].

Equation (7.88) becomes Equation (7.87) when h is infinite, as the complementary error function has a value of zero when the argument of the function is very large. Surface conductance plays a role along with the thermal conductivity in establishing if a body will exhibit a thick body response. Schneider (1973) defined a critical Fourier number when a body ceases to exhibit a thick body response:

$$Fo_{critical} = 0.00756\ Bi^{-0.3} + 0.02 \quad \text{for } 0.001 \leq Bi \leq 1000$$

The Fourier number is: $Fo = \alpha t/(L)^2$; L = thickness/2; Bi = Biot number = hL/k.

Example 7.18. A beef carcass at 38°C is introduced into a cold room at 5°C. The thickness of the carcass is 20 cm. Calculate the temperature at a point 2 cm from the surface after 20 min. The density is 1042 kg/m^3, the thermal conductivity is 0.44 W/(m · K), specific heat is 3558 J/(kg · K), and the surface heat transfer coefficient is 20 W/(m^2· K).

Solution:

For the short exposure time, the material might exhibit a thick body response. This condition will be affirmed by calculating the Biot number and the Fourier number.

$$L = 20(.5) = 10 \text{ cm} = 0.10 \text{ m}$$

$$Bi = \frac{hL}{K} = \frac{20(0.10)}{0.44} = 4.55$$

$$Fo = \frac{\alpha t}{L^2} = \frac{0.44}{1042(3558)}\frac{20(60)}{(0.10)^2} = 0.0142$$

$$Fo_{critical} = 0.00756(4.55)^{-0.3} + 0.02 = 0.024$$

The actual Fourier number is less than the critical value for the solid to cease exhibiting a thick body response, therefore, the error function solution can be used to calculate the temperature at the designated point. Using equation 88, x = 0.02 m from the surface.

$$\alpha = \frac{0.44}{(1042)(3558)} = 1.187 \times 10^{-7}$$

$$\frac{x}{(2)\sqrt{(\alpha t)}} = \frac{0.02}{(2)[(1.187 \times 10^{-7})(20)(60)]^{0.5}}$$
$$= 0.838$$

$$hx/k = 20(0.02)/0.44 = 0.909$$

$$\left(\frac{h}{k}\right)^2 \alpha t = \left(\frac{20}{0.44}\right)\left(1.187 \times 10^{-7}\right)(20)(60) = 0.294$$
$$= 0.542$$

$$\left(\frac{h}{k}\right)\sqrt{(\alpha t)} = \left(\frac{20}{0.44}\right)\left[(1.187 \times 10^{-7})(20)(60)\right]^{0.5}$$

$$\theta = \text{erf}(0.838) + [e]^{0.909+0.294}[\text{erfc}(0.838 + 0.542)]$$
$$= 0.7663812 + (3.3301)(.059674) = 0.9335$$

For cooling: $\theta = (T - T_m)/(T_O - T_m)$

$T = T_m + \theta(T_o - T_m) = 5 + (0.9335)(38 - 5) = 35.8°C$

7.6.4 The Infinite Slab

This is a slab with thickness 2L extending to infinity at both ends.
When h is infinite:

$$\theta = 2\sum_{n=0}^{\infty}\left[\frac{(-1)^n}{(n+0.5)\pi}[e]^{-(n+0.5)^2(\pi^2\alpha t/L^2)}\right]\left[\cos\left[\frac{(n+0.5)\pi x}{L}\right]\right] \tag{7.89}$$

When h is finite:

$$\theta = 2\sum_{n=1}^{\infty}[e]^{-\delta_n{}^2\alpha t/L^2}\left[\frac{\sin(\delta_n)\cos(\delta_n x/L)}{\delta_n + \sin(\delta_n)\cos(\delta_n)}\right] \tag{7.90}$$

δ_n are the positive roots of the transcendental equation:

$$\delta_n \tan(\delta_n) = \frac{hL}{k}$$

The center (θ_c) and surface (θ_s) temperature for an infinite solid with surface heat transfer is obtained by setting x = 0 and x = L in Equation (7.90):

$$\theta_c = 2\sum_{n=1}^{\infty}\frac{\sin(\delta_n)[e]^{-(\delta_n/L)^2\alpha t}}{\delta_n + \sin(\delta_n)\cos(\delta_n)} \tag{7.91}$$

$$\theta_s = 2\sum_{n=1}^{\infty}\frac{\sin(\delta_n)\cos(\delta_n)[e]^{-(\delta_n)^2\alpha t}}{\delta_n + \sin(\delta_n)\cos(\delta_n)} \tag{7.92}$$

7.6.5 Temperature Distribution for a Brick-Shaped Solid

The solution to the differential equation for a brick-shaped solid will be the product of the solution for infinite slabs of dimensions L_1, L_2, and L_3. The roots of the transcendental equation will be different for each dimension of the brick and will be designated δ_{n1}, δ_{n2}, and δ_{n3} respectively, for sides with half thickness L_1, L_2, and L_3.

Let F_o = Fourier number, $\alpha t/L^2$.

$$F(x\delta_{ni}) = \frac{\sin(\delta_{ni})\cos(\delta_{ni}x/L)}{\delta_{ni} + \sin(\delta_{ni})\cos(\delta_{ni})}$$

$$\theta = 8\sum_{n=1}^{\infty}F(x\delta_{n1})F(x\delta_{n2})F(x\delta_{n3})[e]^{-\sum(\delta_{ni})^2 Fo_i} \tag{7.93}$$

Of interest to food scientists and engineers will be center temperature and surface temperature. Let:

$$F(c\delta_{ni}) = \frac{\sin(\delta_{ni})}{\delta_{ni} + \sin(\delta_{ni})\cos(\delta_{ni})}$$

$$F(s\delta_{ni}) = \frac{\sin(\delta_{ni})\cos(\delta_{ni})}{\delta_{ni} + \sin(\delta_{ni})\cos(\delta_{ni})}$$

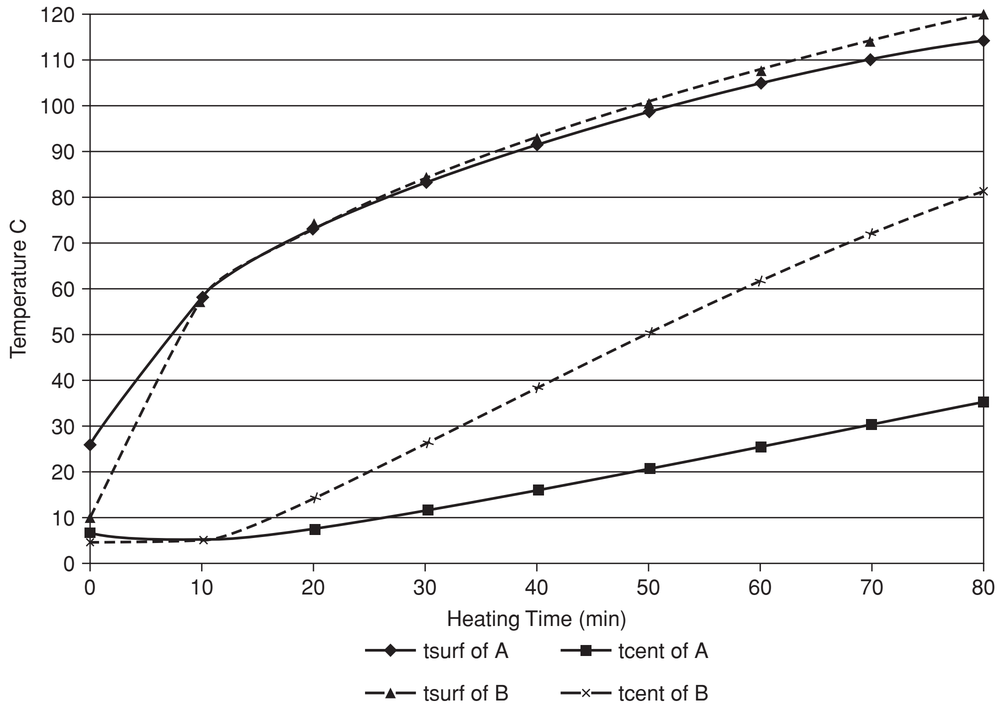

Figure 7.17 Temperature at the surface and at the geometric center of two brick-shaped solids heated in an oven at 177°C from an initial temperature of 4°C. Solid A dimensions: 20.32 × 10.16 × 5.08 cm. Solid B dimensions: 30.48 × 15.24 × 5.08 cm. Parameters: $\rho = 1085$ kg/m^3; $C_p = 4100$ J/(kg · K); k = 0.455 W/(m·K); h = 125 W/(m^2 K).

The center or surface temperature may be calculated using $F(c\delta_{ni})$ or $F(s\delta_{ni})$ in place of $F(x\delta_{ni})$ in Equation (7.93). Appendix Table A.13 lists a computer program in Visual BASIC for calculating the surface and center temperature of a brick shaped solid. An example of the output of the program is shown in Fig 7.17. The surface temperature will be affected by the thickness of the solid and will not assume the heating medium temperature immediately after the start of the heating process.

7.6.6 Use of Heissler and Gurney-Lurie Charts

Before the age of personal computers, calculations involving the transient temperature response of solids was a very laborious process. Solutions to the partial differential equations were plotted and arranged in a form that makes it easy to obtain solutions. Two of the transient temperature charts are the Gurney-Lurie chart shown in Fig. 7.18 and the Heissler chart shown in Figs. 7.19 and 7.20. These charts are for an infinite slab. Their use for a brick-shaped solid will involve the multiplicative superposition technique discussed earlier. Similar charts are available for cylinders and spheres.

Both charts are plots of the dimensionless temperature ratio against the Fourier number. The Gurney-Lurie chart plots the dimensionless position (x/L = n; n = 0 at the center), and 1/Biot number = m, as parameters. The Heissler chart is good for low Fourier numbers, but a chart must be made available for

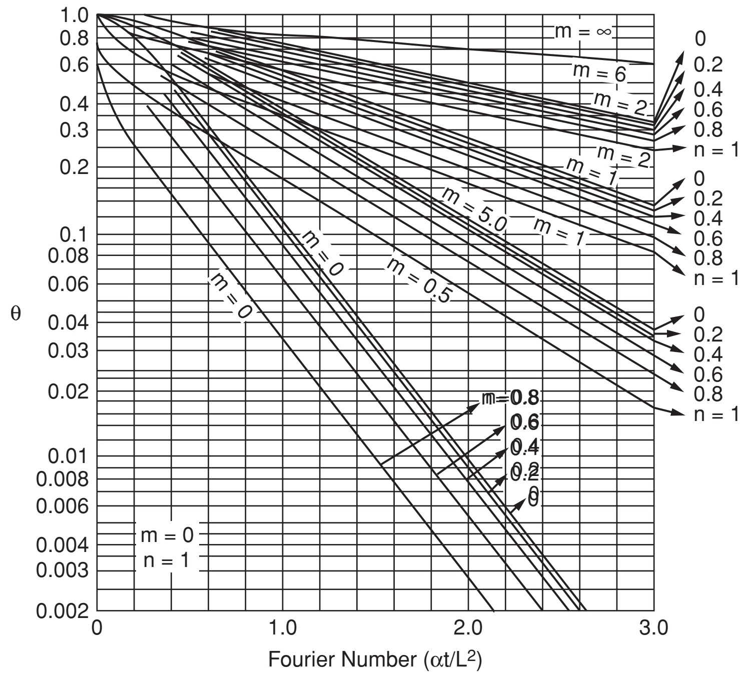

Figure 7.18 Gurney-Lurie chart for the temperature response of an infinite slab. (Source: Adapted from McAdams, W. H. 1954. Heat transmission. 3rd. ed. McGraw-Hill, New York. Used with permission of McGraw-Hill, Inc.)

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each position under consideration. The Heissler chart shown in Fig. 7.19 is the temperature response at the surface and Fig. 7.20 is the temperature response at the center of an infinite slab.

Use of these charts involves calculating the Biot number and the Fourier number. The appropriate curve is then selected which corresponds to the Biot number and the position under consideration. The charts are used to obtain a value for the dimensionless temperature ratio, θ. If the solid is brick shaped, Fourier and Biot numbers are obtained for each direction, and the values of θ obtained for each direction are multiplied to obtain the net temperature response from all three directions.

Example 7.19. Calculate the temperature at the center of a piece of beef 3 cm thick, 12 cm wide, and 20 cm long after 30 minutes of heating in an oven. The beef has a thermal conductivity of 0.45 W/(K), a density of $1008 kg/m^3$, and a specific heat of $3225 J/(kg \cdot K)$. Assume a surface heat transfer coefficient of 20 $W/(m^2 \cdot K)$. The meat was originally at a uniform temperature of 5°C and it was instantaneously placed in an oven maintained at 135°C. Neglect radiant heat transfer and assume no surface evaporation.

Solution:

Consider each side separately. Consider the x direction as the thickness (L = 0.5(0.03) = 0.015 m), the y direction as the width (L = 0.5(0.12) = 0.06 m), and the z direction as the length (L = 0.5(0.2) = 0.1 m).

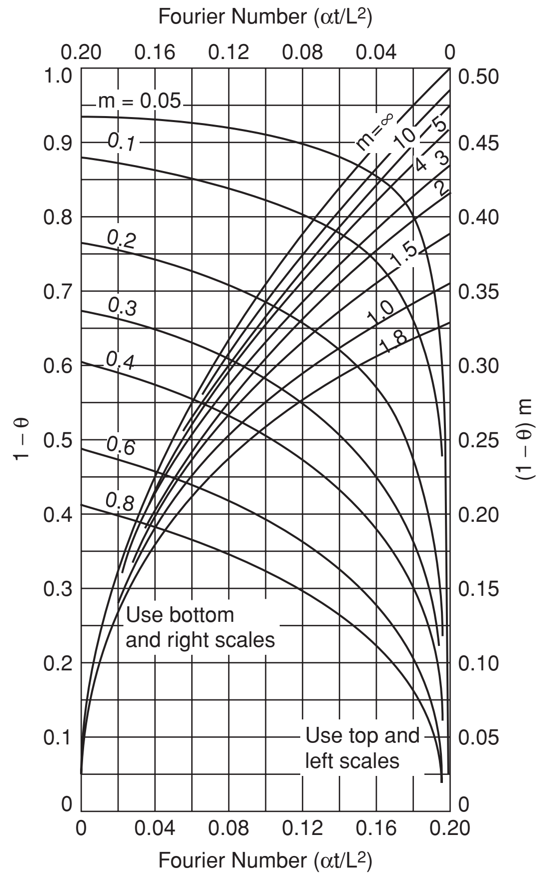

Figure 7.19 Heisler chart for the center temperature of a slab. (Adapted from Hsu, S .T. 1963. Engineering Heat Transfer. Van Nostrand Reinhold, New York.)

$$\alpha = \frac{k}{\rho C_p} = \frac{0.45}{(1008)(3225)} = 1.3843 \times 10^{-7} m^2/s$$

$$Fo_x = [1.3843 \times 10^{-7}(30)(60)]/(0.015)^2 = 1.107$$

$$Fo_y = [1.3843 \times 10^{-7}(30)(60)]/(0.06)^2 = 0.0692$$

$$Fo = \frac{\alpha t}{L^2}$$

$$Fo_z = [1.3843 \times 10^{-7}(30)(60)]/(0.1)^2 = 0.249$$

$$Bi = hL/k$$

$$Bi_x = 20(0.015)/0.45 = 0.667; m = 1.5$$

$$Bi_y = 20(0.06)/0.45 = 2.667; m = 0.374$$

$$Bi_z = 20(0.1)/0.45 = 4.444; m = 0.225$$

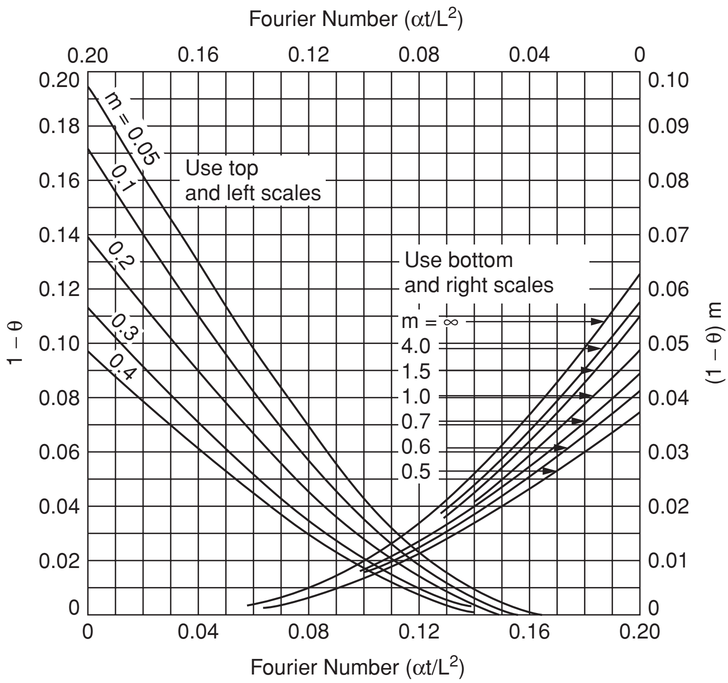

Figure 7.20 Heisler chart for the surface temperature of a slab. (Adapted from Hsu, S. T. 1963. Engineering Heat Transfer. Van Nostrand Reinhold, New York.)

At the center, $n = 0$

From Fig. 7.18, θ for $m = 1.5$ will be obtained by interpolation between values for $m = 1$ and $m = 2$. From Fig. 7.18, $Fo_x = 1.107$, $n = 0$, $m = 1$, $\theta = 0.52$, $n = 0$, $m = 2$, $\theta = 0.7$. For $m = 1.5$, $\theta = [0.52 + (0.70 - 0.52)(1.5 - 1)]/1 = 0.61$

The values of θ for $Fo_y = 0.0692$ and Fo_z appears to be almost 1.0 from Fig. 7.18. To verify, use Fig. 7.20 to see that at $m = 0.4$ and $Fo = 0.07$, $1 - \theta = 0$, and at $m = 0.2$ amd $Fo = 0.01$, $1 - \theta = 0$. Thus, heating for this material occurs primarily from one dimension.

$$T = T_m - \theta(T_m - T_o)$$

$$= 135 - 0.61(135 - 5) = 55.7°C.$$

7.7 CALCULATING SURFACE HEAT TRANSFER COEFFICIENTS FROM EXPERIMENTAL HEATING CURVES

If heating proceeds for a long time, the series represented by Equation (7.93) converges rapidly and the first term in the series is adequate. Equation (7.93) then becomes:

$$\theta_c = 2F(c\delta_1)F(c\delta_2)F(c\delta_3)[e]^{-Fo_1\delta_1{}^2 - Fo_2\delta_2{}^2 - Fo_3\delta_3{}^2}$$

Let $F(c\delta) = 2F(c\delta_1)F(c\delta_2)F(c\delta_3)$. Taking the logarithms of both sides of the equation:

$$(\theta_c) = \log[F(C\delta)] - [\alpha t \log(e)]\left[\frac{\delta_1{}^2}{L_1{}^2} + \frac{\delta_2{}^2}{L_2{}^2} + \frac{\delta_3{}^2}{L_3{}^2}\right] \tag{7.94}$$

Equation (7.94) shows that a plot of $\log(\theta_c)$ against t will be linear, and the surface heat transfer coefficient can be determined from the slope if α is known.

$$\text{Slope} = -[\alpha \log(e)]\left[\frac{\delta_1{}^2}{L_1{}^2} + \frac{\delta_2{}^2}{L_2{}^2} + \frac{\delta_3{}^2}{L_3{}^2}\right] \tag{7.95}$$

A computer program in BASIC shown in Appendix Table A. 14 can be used to determine the average heat transfer coefficient from the center temperature heating curve of a brick shaped solid. Similar approaches may be used for cylindrical solids.

7.8 FREEZING RATES

Temperature distribution in solids exposed to a heat exchange medium at temperatures below the solid's freezing point is complicated by the change in phase and the unique properties of the frozen and unfrozen zones in the solid. In addition, the ice front advances towards the interior from the surface, and at the interface between the two zones, a tremendous heat sink exists, in the form of the heat of fusion of water. A number of approaches have been used to mathematically model the freezing process, but the most successful in terms of simplicity and accuracy is the refinement by Cleland and Earle (1984) of the empirical equation originally developed by Plank (1913).

Plank's original equation was

$$t_f = \frac{\lambda}{T_f - T_a}\left[\frac{PD}{h} + \frac{RD^2}{k}\right] \tag{7.96}$$

where t_f is freezing time for a solid with a freezing point T_f and a thermal thickness D, which is the full thickness in the case of a slab or the diameter in the case of a sphere. P and R are shape constants ($P = 1/6$ for a sphere, 1/4 for a cylinder, and 1/2 for a slab; $R = 1/24$ for a sphere, 1/16 for a cylinder, and 1/8 for a slab). G is the latent heat of fusion per unit volume, h is heat transfer coefficient, and k is thermal conductivity of the frozen solid. Shape factors P and R have been developed for a brick shaped solid and presented as a graph based on the dimensions of the brick. This graph can be seen in Charm (1971). Plank's equation has been refined by Cleland and Earle (1984) to account for the fact that not all water freezes at the freezing point, and that the freezing process may proceed to specific final product temperature. The effect of an initial temperature different from the freezing point has also been included in the analysis. Cleland and Earle's (1984) equation is

$$t_f = \frac{\Delta H_{10}}{(T_f - T_a)(\text{EHTD})}\left[P\frac{D}{h} + R\frac{D^2}{k_s}\right]\left[1 - \frac{1.65\,\text{STE}}{k_s}\ln\left(\frac{T_{fin} - T_a}{-10 - T_a}\right)\right] \tag{7.97}$$

ΔH_{10} = enthalpy change to go from T_f to $-10°C$ in J/m^3

T_a = freezing medium temperature

T_{fin} = final temperature

EHTD = equivalent heat transfer dimensionality, defined as the ratio of the time to freeze a slab of half thickness, D, to the time required to freeze the solid having the same D. EHTD = 1 for a slab with large width and length such that heat transfer is effectively only in one direction, and 3 for a sphere. Bricks and cylinders will have EHTD between 1 and 3.

h = surface heat transfer coefficient

k_s = thermal conductivity of the frozen solid

P, R = parameters that are functions of the Stephan number (STE) and the Plank number (PK)

$$STE = C_s \frac{(T_f - T_a)}{\Delta H_{10}}; \quad PK = C_1 \frac{(T_i - T_f)}{\Delta H_{10}}$$

C_s = volumetric heat capacity of frozen material, $J/(m^3 \cdot K)$

C_1 = volumetric heat capacity of unfrozen material, $J/(m^3 \cdot K)$

T_i = initial temperature of solid

$P = 0.5\,[1.026 + 0.5808\,PK + STE(0.2296\,PK + 0.105)]$

$R = 0.125\,[1.202 + STE(3.41\,PK + 0.7336)]$

The major problem in using Equatiion (7.97) is the determination of EHTD. EHTD can be determined experimentally by freezing a slab and the solid of interest and taking the ratio of the freezing times. EHTD considers the total dimensionality of the material instead of just one dimension.

Example 7.20. Calculate the freezing time for blueberries in a belt freezer where the cooling air is at −35°C. The blueberries have a diameter of 0.8 cm and are to be frozen from an initial temperature of 15°C to a final temperature of −20°C. They contain 10% soluble solids, 1% insoluble solids, and 89% water. Use Choi and Okos' (1987) correlation for determining the thermal conductivity, Chang and Tao's (1981) correlation for determining the freezing point and enthalpy change below the freezing point, and Seibel's equation for determining the specific heat above and below freezing. The blueberries have a density of 1070 kg/m^3 unfrozen and 1050kg/m^3 frozen. The heat transfer coefficient is 120 $W/(m^3 \cdot K)$.

Solution:

For the specific heat above freezing, use Equation (5.11), Chapter 5:

$$C_1' = 837.36(0.11) + 4186.8(0.89) = 3818.3\,J/(kg.K)$$

Converting to J/(K):

$$C_1 = \frac{3818\,J}{kg\,K}\frac{1070\,kg}{m^3} = 4.0856 \times 10^6\,J/(m^3 \cdot K)$$

For the specific heat below freezing, use equation 13, Chapter 5:

$$C_s' = 837.36(0.11) + 2093.4(0.89) = 1955.63\,J/(kg \cdot K)$$

Converting to $J/(m^3 \cdot K)$:

$$C_s = \frac{1955.63\,J}{kg \cdot K}\frac{1050\,kg}{m^3} = 2.053 \times 10^6\,J/(m^3\,K)$$

Use Chang and Tao's correlation for the enthalpy change fetween freezing point and −10°C (see Chapter 5, the section "Enthalpy Change with a Change in Phase"). From Equation (5.20), Chapter 5:

$$T_f = 287.56 - 49.19(0.89) + 37.07(0.89)^2 = 273\ K$$

Solving for the enthalpy at the freezing point: Equation (5.22), Chapter 5: $H_f = 9792.46 + 405096(0.89)$

$$H_f = 370327.9\ \text{J/kg}$$
$$T = -10°\text{C} = 263\ \text{K}$$
$$T_r = (263 - 227.6)/(273 - 227.6) = 0.776$$
$$H = H_f\ [\alpha T_r + (1 - a)(T_r)^b]$$

Solving for α: Equation (5.17), Chapter 5.

$$\alpha = 0.362 + 0.0498(0.89 - 0.73) - 3.465(0.89 - 0.73)^2$$
$$= 0.362 + 0.007968 - 0.088704 = 0.281$$

Solving for b: Equation (5.18), Chapter 5:

$$b = 27.2 - 129.04(0.89 - 0.23) - 481.46(0.89 - 0.23)^2$$
$$= 27.2 - 6.6151 - 1.265276 = 19.3196$$

Solving for H at $-10°$C, $T_r = 0.776$; Equation (5.23), Chapter 5:

$$H = H_f \left[0.281)(0.776) + (1 - 0.281)(0.776)^{19.3196}\right]$$
$$= 0.2234\,H_f; \quad \Delta H_{10} = H_f(1 - 0.2234) = 0.7766(370327.9)$$
$$\Delta H_{10} = 2.876 \times 105\,\text{J/kg}$$

The thermal conductivity will be determined using the spreadsheet program (Fig. 7.1) in the section "Estimation of Thermal Conductivity of Food Products." The amount of water unfrozen at the freezing point will be determined from the enthalpy at the freezing point using the base temperature for Chang and Tao's enthalpy correlations, of 227.3 K as the reference, and assuming that all the water is in the form of ice. The heat of fusion of ice is 334,944 J/kg.

The specific heat below freezing calculated above was 1955.23 J/(kg · K)

$$H_f' = 0.89(334, 944) + 1955.23(273 - 227.6) = 386, 867$$

Mass fraction unfrozen water at the freezing point

$$= (386{,}867 - 370{,}327.9)/334{,}944 = 0.049$$

Using the BASIC program in Appendix Table A.11, the following are entered with the prompts: $X_{water} = 0.049$; $X_{ice} = 0.89 - 0.049 = 0.8406$; $X_{carb} = 0.10$; $X_{fiber} = 0.01$; all other components are zero. $T = -10°$C; The output is: $k_s = 2.067\text{W}/(\text{m} \cdot \text{K})$. Solving for freezing time using Equation (7.97):

$$\Delta H_{10} = 2.876 \times 105\,\text{J/kg}\ (1070\,\text{kg/m}^3) = 3.077 \times 108\,\text{J/m}^3$$
$$T_a = -35°\text{C};\ T_{fin} = -20°\text{C}$$
$$h = 120\,\text{W}/(\text{m}^2 \cdot \text{K});\ \text{EHDT} = 3 \text{ for a sphere};\ D = 0.008\,\text{m}$$
$$k_s = 2.067\,\text{W}/(\text{m} \cdot \text{K}); T_i = 15\,\text{EC};\ C_s = 2.053 \times 10^6\,\text{J/m}^3$$
$$C_1 = 4.0856\text{J}/(\text{m}^3\ \text{AK})$$
$$\text{STE} = 2.053 \times 10^6 \left[\frac{0 - (-35)}{6.685 \times 10^7}\right] = 1.0749$$

$$PK = 4.0856 \times 10^6 \left[\frac{15 - 0}{6.685 \times 10^7} \right] = 0.9167$$

$$\begin{aligned} P &= 0.5[1.026 + 0.5808(0.9167) \\ &\quad + 1.0749[0.2296(0.9167) + 0.105] = 0.9488 \\ R &= 0.125[1.202 + 1.0749[(3.41)(0.91674) + 0.7336]] = 0.6688 \end{aligned}$$

Substituting values in Equation (7.97):

$$\begin{aligned} t_f &= \frac{3.077 \times 10^8}{[0 - (-35)](3)} \left[0.9488 \frac{0.008}{120} + 0.6688 \frac{(0.008)^2}{2.067} \right] \\ &= 0.029395 \times 10^8 [0.00006325 + 0.00002071](1.4383) = 353.9\text{s} \\ &\quad \left[1 - \frac{1.65(1.0749)}{2.067} \ln \left[\frac{-20 - (-35)}{-10 - (-35)} \right] \right] \end{aligned}$$

PROBLEMS

7.1. How many inches of insulation would be required to insulate a ceiling such that the surface temperature of the ceiling facing the living area is within 2°C of the room air temperature. Assume a heat transfer coefficient on both sides of the ceiling of 2.84 W/($m^2 \cdot$ K) and a thermal conductivity of 0.0346 W/(m · K) for the insulation. The ceiling is 1.27-cm-thick plasterboard with a thermal conductivity of 0.433 W/(m · K). Room temperature is 20°C and attic temperature is 49°C.

7.2. If a heat transfer coefficient of 2.84 W/($m^2 \cdot$ K) exists on each of the two inside faces of 6.35-mm-thick glass separated by an air gap, calculate the gap that could be used such that the rate of heat transfer by conduction through the air gap would equal the rate of heat transfer by convection. What would be the rate of heat transfer if this gap is exceeded?

The thermal conductivity of air is 0.0242 W/(m · K). Solve for temperatures of 20°C and −12°C on the outside surfaces of the glass. Would the calculated gap change in value for different values of the surface temperatures? Would there be any advantage to increasing the gap beyond this calculated value?

7.3. A walk-in freezer 4 × 6 m and 3 m high is to be built. The walls and ceiling consist of 1.7 mm thick stainless steel (k = 14.2 W/(m · K)), 10 cm thick of foam insulation (k = 0.34 W/(m · K)), a thickness of corkboard (k = 0.043 W/(m · K)), and 1.27 cm thick of wood siding (k = 0.43 W/(m · K)). The inside of the freezer is maintained at −40°C. Ambient air outside the freezer is at 32°C. The heat transfer coefficient is 5 W/($m^2 \cdot$ K) on the wood side of the wall, and 2 W/($m^2 \cdot$ K) on the stainless steel side.

(a) If the outside air has a dew point of 29°C, calculate the thickness of the corkboard insulation that would prevent condensation of moisture on the outside wall of this freezer.

(b) Calculate the rate of heat transfer through the walls and ceiling of this freezer.

7.4. (a) Calculate the rate of heat loss to the surroundings and the quantity of steam that would condense per hour per meter of a l.5-in. (nominal) schedule 40 steel pipe containing steam at 130°C. The heat transfer coefficient on the steam side is 11,400 W/($m^2 \cdot$ K) and on the outside of the pipe to air is 5.7 W/$m^2 \cdot$ K. Ambient air averages 15°C in temperature for the year. The thermal conductivity of the steel pipe wall is 45 W/(m · K).

(b) How must energy would be saved in one year (365 days, 24 h/day) if the pipe is insulated with 5-cm-thick insulation having a thermal conductivity of 0.07 W/(m · K). The heat transfer coefficients on the steam and air sides are the same as in (a).

7.5. The rate of insolation (solar energy impinging on a surface) on a solar collector is 475W/m^2. Assuming that 80% of the impinging radiation is absorbed (the rest is reflected), calculate the rate at which water can be heated from 15°C to 50°C in a solar hot water heater consisting of a spiral-wound, horizontal, 1.9-cm inside-diameter polyethylene pipe 30 m long. The pipe has a wall thickness of 1.59 mm. The projected area of a horizontal cylinder receiving radiation is the diameter multiplied by the length. Assume a heat transfer coefficient of 570 W/(m^2· K) on the water side and a heat transfer coefficient of 5 W/(m^2· K) on the air side, when calculating heat loss to the surroundings after the radiant energy from the sun is absorbed. The thermal conductivity of the pipe wall is 0.3 W/m · K. Ambient temperature is 7°C.

7.6. A swept surface heat exchanger cools 3700 kg of tomato paste per hour from 93°C to 32°C. If the overall heat transfer coefficient based on the inside surface area is 855 W/m^2 · K, calculate the heating surface area required for concurrent flow and countercurrent flow. Cooling water enters at 21°C and leaves at 27°C. The specific heat of tomato paste is 3560 J/(kg · K).

7.7. Design the heating and cooling section of an aseptic canning system that processes 190 L per minute of an ice cream mix. The material has a density of 1040 kg/m^3 and a specific heat of 3684 J/(kg · K).

(a) Calculate the number of units of swept surface heat exchangers required to heat the material from 39°C to 132°C. Each unit has an inside heat transfer surface area of 0.97m^2. The heating medium is steam at 143°C. Previous experience with a similar unit on this material was that an overall heat transfer coefficient of 1700 W/(m^2 · K) based on the inside surface area may be expected.

(b) Calculate the number of units (0.9 m^2 inside surface area per unit) required for cooling the sterilized ice cream mix from 132°C to 32°C. The cooling jacket of the swept surface heat exchangers is cooled by freon refrigerant from a refrigeration system at −7°C. Under these conditions, a heat transfer coefficient of 855 W/(m^2 · K) based on the inside surface area may be expected.

7.8. A small swept surface heat exchanger having an inside heat transfer surface area of 0.11 m^2 is used to test the feasibility of cooking a slurry in a continuous system. When the slurry was passed through the heat exchanger at a rate of 168 kg/h it was heated from 25°C to 72°C. Steam at 110°C was used. The slurry has a specific heat of 3700 J/(kg · K).

(a) What is the overall heat transfer coefficient in this system?

(b) If this same heat transfer coefficient is expected in a larger system, calculate the rate at which the slurry can be passed through a similar swept surface heat exchanger having a heat transfer surface area inside of 0.75 m^2, if the inlet and exit temperatures are 25°C and 72°C, respectively, and steam at 120°C is used for heating.

7.9. A steam jacketed kettle has an inside heat transfer surface area of 0.43 m^2 that is all completely covered by the product. The product needs to be heated from 10°C to 99°C.
The product contains 80% water and 20% nonfat solids. Previous experience has established that an overall heat transfer coefficient based on the inside surface area of 900 W/(m^2 · K) may be expected. The kettle holds 50 kg of product. Condensing steam at 120°C is in the heating jacket. The contents are well stirred continuously during the process.

(a) Calculate the time required for the heating process to be completed.

(b) Determine the nearest nominal size steel pipe that can be used to supply steam to this kettle if the rate of steam flow through the pipe is to average a velocity of 12 m/s.

7.10. A processing line for a food product is being designed. It is necessary to estimate the number of kettles that is required to provide a production capacity of 500 kg/h. The cooking process involving the kettles requires heating the batch from 27°C to 99°C, simmering at 99°C for 30 minutes and filling the hot product into cans. Filling the kettles and emptying requires approximately 15 minutes. The specific heat of the product is 3350 J/(kg · K). The density is 992 kg/m^3.

Available for heating are cylindrical vessels with hemispherical bottoms with the hemisphere completely jacketed. The height of the cylindrical section is 25 cm. The diameter of the vessel is 0.656 m. Assume the vessels are filled to 85% of capacity each time. The overall heat transfer coefficient based on the inside surface area averages 600 W/(m^2 · K). How many kettles are required to provide the desired production capacity? Steam condensing at 120°C is used in the jacket for heating.

7.11. A process for producing frozen egg granules is proposed where a refrigerated rotating drum that has a surface temperature maintained at −40°C contacts a pool of liquid eggs at 5°C. The eggs freeze on the drum surface and the frozen material is scraped off the drum surface at a point before the surface reenters the liquid eggs. The frozen material, if thin enough will be collected as frozen flakes. In this process, the thickness of frozen egg that forms on the drum surface is determined by the dwell time of the drum within the pool of liquid. An analogy of the process, which may be solved using the principles discussed in the section on freezing water, 7.5 h is the freezing of a slab directly in contact with a cold surface (i.e, h is infinite). It is desired that the frozen material be 2 mm thick on the drum surface.

(a) Calculate the dwell time of the drum surface within the liquid egg pool. Assume that on emerging from the liquid egg pool, the frozen material temperature will be 2°C below the freezing point.

(b) If the drum has a diameter of 50 cm and it travels 120E ($^1\!/_3$ of a full rotation) after emerging from the liquid egg pool before the frozen material is scraped off, calculate the rotational speed of the drum needed to satisfy the criterion stipulated in (a), and calculate the average temperature of the frozen material at the time it is scraped off the drum. The density of liquid eggs is 1012 kg/m^3, and frozen eggs 1009 kg/m^3. Calculate the thermophysical properties based on the following compositional data: 75% water, 12% protein, 12% fat, 1% carbohydrates.

7.12. In an experiment for pasteurization of orange juice, an 0.25-in. outside-diameter tube with 1/32-in.-thick wall was made into a coil and immersed in a water bath maintained at 95°C. The coil was 2 m long and when the juice was pumped at the rate of 0.2 L/min, the juice temperature changed from 25°C to 85°C. The juice contained 12% total solids. Calculate:

(a) The overall heat transfer coefficient.

(b) The inside local heat transfer coefficient if the ratio h_o/h_i is 0.8.

(c) The inside local heat transfer coefficient, h_i is directly proportional to the 0.8 power of the average velocity. If the rate at which the juice is pumped through the system is increased to 0.6 L/min, calculate the tube length needed to raise the temperature from 25°C to 90°C.

7.13. Calculate the surface area of a heat exchanger needed to pasteurize 100 kg/h of catsup by heating in a one-pass shell and tube heat exchanger from 40°C to 95°C. The catsup density is 1090 kg/m^3. The flow behavior index is 0.5, and the consistency index is 0.5 and 0.35 Pa ·s^n at 25°C and 50°C, respectively. Estimate the thermal conductivity and specific heat using correlations discussed in Chapters 5 and 7. The catsup contains 0.4% fiber, 33.8% carbohydrate, 2.8% ash, and the balance is water. The catsup is to travel within the heat exchanger at a velocity of 0.3 m/s. Steam condensing at 135°C is used for heating and, a heat transfer coefficient of

$15,000 W/(m^2 \cdot K)$ may be assumed for the steam side. The heat exchanger tubes are type 304 stainless steel with an inside diameter of 0.02291 m and an outside diameter of 0.0254 m.

7.14. A box of beef that was tightly packed was originally at 0°C. It was inadvertently left on a loading dock during the summer when ambient conditions were 30°C. Assuming that the heat transfer coefficient around the box averages $20 W/(m^2 \cdot K)$, calculate the surface temperature, and the temperature at a point 2 cm deep from the surface after 2 hours. The box consisted of 0.5-cm-thick fiberboard with a thermal conductivity of $0.2 W/(m \cdot K)$, and had dimensions of $47 \times 60 \times 30$ cm. Because the box material has very little heat capacity, it may be assumed to act as a surface resistance, and an equivalent heat transfer coefficient may be calculated such that the resistance to heat transfer will be the same as the combined conductive resistance of the cardboard and the convective resistance of the surface heat transfer coefficient. The meat has a density of $1042 kg/m^3$, a thermal conductivity of $0.44 W/(m \cdot K)$, and a specific heat of $3558 J/(kg \cdot K)$.

7.15. In the operation of a microwave oven, reduced power application to the food is achieved by alternatively cutting on and off the power applied. To minimize excessive heating in some parts of the food being heated, the power application must be cycled such that the average temperature rise with each application of power does not exceed 5°C followed by a 10-second pause between power application. If 0.5 kg of food is being heated in a microwave oven having a power output of 600 W, calculate fraction of full power output that must be set on the oven controls such that the above temperature rise and pause cycles are satisfied. The food has a specific heat of $3500 J/(kg \cdot K)$.

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